6-6

Study Guide

Modeling Real-World Data with Sinusoidal Functions

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan. Write a sinusoidal function that models the average monthly temperatures, using $t = 1$ to represent January. Temperatures are in degrees Fahrenheit (°F).**

Jan.	30°
Feb.	34°
Mar.	45°
Apr.	59°
May	71°
June	80°
July	84°
Aug.	81°
Sept.	74°
Oct.	62°
Nov.	48°
Dec.	35°

These data can be modeled by a function of the form $y = A \sin (kt + c) + h$, where t is the time in months.

First, find A, h, and k.

A: $A = \frac{84 - 30}{2}$ or 27 — *A is half the difference between the greatest temperature and the least temperature.*

h: $h = \frac{84 + 30}{2}$ or 57 — *h is half the sum of the greatest value and the least value.*

k: $\frac{2\pi}{k} = 12$ — *The period is 12.*

$k = \frac{\pi}{6}$

Substitute these values into the general form of the function.

$y = A \sin (kt + c) + h \qquad y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$

To compute c, substitute one of the coordinate pairs into the equation.

$$y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$$

$$30 = 27 \sin \left[\frac{\pi}{6}(1) + c\right] + 57 \qquad \textit{Use } (t, y) = (1, 30).$$

$$-27 = 27 \sin \left(\frac{\pi}{6} + c\right) \qquad \textit{Subtract 57 from each side.}$$

$$-\frac{27}{27} = \sin \left(\frac{\pi}{6} + c\right) \qquad \textit{Divide each side by 27.}$$

$$\sin^{-1} (-1) = \frac{\pi}{6} + c \qquad \textit{Definition of inverse}$$

$$\sin^{-1} (-1) - \frac{\pi}{6} = c \qquad \textit{Subtract } \frac{\pi}{6} \textit{ from each side.}$$

$$-2.094395102 \approx c \qquad \textit{Use a calculator.}$$

The function $y = 27 \sin \left(\frac{\pi}{6}t - 2.09\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.