11-6

Study Guide

Natural Logarithms

Logarithms with base e are called **natural logarithms** and are usually written **ln x.** Logarithms with a base other than e can be converted to natural logarithms using the change of base formula. The antilogarithm of a natural logarithm is written **antiln x.** You can use the properties of logarithms and antilogarithms to simplify and solve exponential and logarithmic equations or inequalities with natural logarithms.

Example 1 **Convert $\log_4 381$ to a natural logarithm and evaluate.**

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_4 381 = \frac{\log_e 381}{\log_e 4} \qquad a = 4,\ b = e,\ n = 381$$

$$= \frac{\ln 381}{\ln 4} \qquad \log_e x = \ln x$$

$$\approx 4.2868 \qquad \textit{Use a calculator.}$$

So, $\log_4 381$ is about 4.2868.

Example 2 **Solve $3.75 = -7.5 \ln x$.**

$$3.75 = -7.5 \ln x$$

$$-0.5 = \ln x \qquad \textit{Divide each side by } -7.5$$

$$\text{antiln}\,(-0.5) = x \qquad \textit{Take the antilogarithm of each side.}$$

$$0.6065 \approx x \qquad \textit{Use a calculator.}$$

The solution is about 0.6065.

Example 3 **Solve each equation or inequality by using natural logarithms.**

a. $\mathbf{4^{3x} = 6^{x+1}}$

$$4^{3x} = 6^{x+1}$$

$$\ln 4^{3x} = \ln 6^{x+1} \qquad \textit{Take the natural logarithm of each side.}$$

$$3x \ln 4 = (x + 1) \ln 6 \qquad \ln a^n = n \ln a$$

$$3x(1.3863) = (x + 1)(1.7918) \qquad \textit{Use a calculator.}$$

$$4.1589x = 1.7918x + 1.7918$$

$$2.3671x = 1.7918$$

$$x \approx 0.7570$$

b. $\mathbf{25 > e^{0.2t}}$

$$25 > e^{0.2t}$$

$$\ln 25 > \ln e^{0.2t} \qquad \textit{Take the natural logarithm of each side.}$$

$$\ln 25 > 0.2t \ln e \qquad \ln a^n = n \ln a$$

$$3.2189 > 0.2t \qquad \textit{Use a calculator.}$$

$$16.0945 > t$$

Thus, $t < 16.0945$