3-4

Study Guide

Inverse Functions and Relations

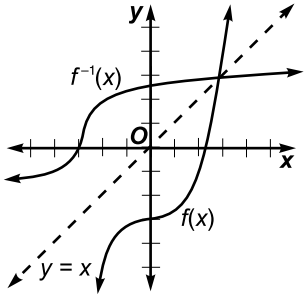
Two relations are inverse relations if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b). If the inverse of the function $f(x)$ is also a function, then the inverse is denoted by $f^{-1}(x)$.

Example 1 **Graph $f(x) = \frac{1}{4}x^3 - 3$ and its inverse.**

To graph the function, let $y = f(x)$. To graph $f^{-1}(x)$, interchange the x- and y-coordinates of the ordered pairs of the function.

$f(x) = \frac{1}{4}x^3 - 3$	
x	y
−3	−9.75
−2	−5
−1	−3.25
0	−3
1	−2.75
2	−1
3	3.75

$f^{-1}(x)$	
x	y
−9.75	−3
−5	−2
−3.25	−1
−3	0
−2.75	1
−1	2
3.75	3

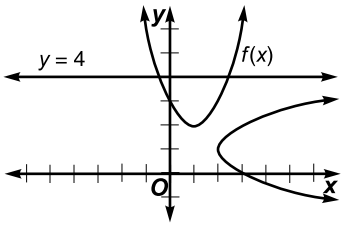


You can use the **horizontal line test** to determine if the inverse of a relation will be a function. If every horizontal line intersects the graph of the relation in at most one point, then the inverse of the relation is a function.

You can find the inverse of a relation algebraically. First, let $y = f(x)$. Then interchange x and y. Finally, solve the resulting equation for y.

Example 2 **Determine if the inverse of $f(x) = (x - 1)^2 + 2$ is a function. Then find the inverse.**

Since the line $y = 4$ intersects the graph of $f(x)$ at more than one point, the function fails the horizontal line test. Thus, the inverse of $f(x)$ is not a function.



$y = (x - 1)^2 + 2$	*Let $y = f(x)$.*
$x = (y - 1)^2 + 2$	*Interchange x and y.*
$x - 2 = (y - 1)^2$	*Isolate the expression containing y.*
$\pm\sqrt{x - 2} = y - 1$	*Take the square root of each side.*
$y = 1 \pm \sqrt{x - 2}$	*Solve for y.*