7-5

Study Guide

Solving Trigonometric Equations

When you solve trigonometric equations for **principal values** of x, x is in the interval $-90° \le x \le 90°$ for $\sin x$ and $\tan x$. For $\cos x$, x is in the interval $0° \le x \le 180°$. If an equation cannot be solved easily by factoring, try writing the expressions in terms of only one trigonometric function.

Example 1 **Solve $\tan x \cos x - \cos x = 0$ for principal values of x. Express solutions in degrees.**

$$\begin{aligned} \tan x \cos x - \cos x &= 0 \\ \cos x (\tan x - 1) &= 0 && \textit{Factor.} \\ \cos x = 0 \quad \text{or} \quad \tan x - 1 &= 0 && \textit{Set each factor equal to 0.} \\ x = 90° \qquad \tan x &= 1 \\ x &= 45° \end{aligned}$$

When $x = 90°$, $\tan x$ is undefined, so the only principal value is 45°.

Example 2 **Solve $2 \tan^2 x - \sec^2 x + 3 = 1 - 2 \tan x$ for $0 \le x < 2\pi$.**

This equation can be written in terms of $\tan x$ only.

$$\begin{aligned} 2 \tan^2 x - \sec^2 x + 3 &= 1 - 2 \tan x \\ 2 \tan^2 x - (\tan^2 x + 1) + 3 &= 1 - 2 \tan x && \textit{sec}^2\, x = \textit{tan}^2\, x + 1 \\ \tan^2 x + 2 &= 1 - 2 \tan x && \textit{Simplify.} \\ \tan^2 x + 2 \tan x + 1 &= 0 \\ (\tan x + 1)^2 &= 0 && \textit{Factor.} \\ \tan x + 1 &= 0 && \textit{Take the square root of each side.} \\ \tan x &= -1 \\ x &= \frac{3\pi}{4} \text{ or } x = \frac{7\pi}{4} \end{aligned}$$

When you solve for all values of x, the solution should be represented as $x + 360°k$ or $x + 2\pi k$ for $\sin x$ and $\cos x$ and $x + 180°k$ or $x + \pi k$ for $\tan x$, where k is any integer.

Example 3 **Solve $\sin x + \sqrt{3} = -\sin x$ for all real values of x.**

$$\begin{aligned} \sin x + \sqrt{3} &= -\sin x \\ 2 \sin x + \sqrt{3} &= 0 \\ 2 \sin x &= -\sqrt{3} \\ \sin x &= -\frac{\sqrt{3}}{2} \\ x &= \frac{4\pi}{3} + 2\pi k \text{ or } x = \frac{5\pi}{3} + 2\pi k, \text{ where } k \text{ is any integer} \end{aligned}$$

The solutions are $\frac{4\pi}{3} + 2\pi k$ and $\frac{5\pi}{3} + 2\pi k$.