8-4

Study Guide

Perpendicular Vectors

Two vectors are perpendicular if and only if their **inner product** is zero.

Example 1 **Find each inner product if $\vec{u} = \langle 5, 1\rangle$, $\vec{v} = \langle -3, 15\rangle$, and $\vec{w} = \langle 2, -1\rangle$. Is either pair of vectors perpendicular?**

a. $\vec{u} \cdot \vec{v}$

$$\begin{aligned}\vec{u} \cdot \vec{v} &= 5(-3) + 1(15)\\ &= -15 + 15\\ &= 0\end{aligned}$$

$\vec{u}$ and $\vec{v}$ are perpendicular.

b. $\vec{v} \cdot \vec{w}$

$$\begin{aligned}\vec{v} \cdot \vec{w} &= -3(2) + 15(-1)\\ &= -6 + (-15)\\ &= -21\end{aligned}$$

$\vec{v}$ and $\vec{w}$ are not perpendicular.

Example 2 **Find the inner product of $\vec{r}$ and $\vec{s}$ if $\vec{r} = \langle 3, -1, 0\rangle$ and $\vec{s} = \langle 2, 6, 4\rangle$. Are $\vec{r}$ and $\vec{s}$ perpendicular?**

$$\begin{aligned}\vec{r} \cdot \vec{s} &= (3)(2) + (-1)(6) + (0)(4)\\ &= 6 + (-6) + 0\\ &= 0\end{aligned}$$

$\vec{r}$ and $\vec{s}$ are perpendicular since their inner product is zero.

Unlike the inner product, the **cross product** of two vectors is a vector. This vector does not lie in the plane of the given vectors but is perpendicular to the plane containing the two vectors.

Example 3 **Find the cross product of $\vec{v}$ and $\vec{w}$ if $\vec{v} = \langle 0, 4, 1\rangle$ and $\vec{w} = \langle 0, 1, 3\rangle$. Verify that the resulting vector is perpendicular to $\vec{v}$ and $\vec{w}$.**

$$\begin{aligned}\vec{v} \times \vec{w} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k}\\ 0 & 4 & 1\\ 0 & 1 & 3\end{vmatrix}\\ &= \begin{vmatrix} 4 & 1\\ 1 & 3\end{vmatrix}\vec{i} - \begin{vmatrix} 0 & 1\\ 0 & 3\end{vmatrix}\vec{j} + \begin{vmatrix} 0 & 4\\ 0 & 1\end{vmatrix}\vec{k} \quad \textit{Expand by minors.}\\ &= 11\vec{i} - 0\vec{j} + 0\vec{k}\\ &= 11\vec{i} \text{ or } \langle 11, 0, 0\rangle\end{aligned}$$

Find the inner products.

$\langle 11, 0, 0\rangle \cdot \langle 0, 4, 1\rangle$
$11(0) + 0(4) + 0(1) = 0$

$\langle 11, 0, 0\rangle \cdot \langle 0, 1, 3\rangle$
$11(0) + 0(1) + 0(3) = 0$

Since the inner products are zero, the cross product $\vec{v} \times \vec{w}$ is perpendicular to both $\vec{v}$ and $\vec{w}$.