4-7

Study Guide

Radical Equations and Inequalities

Equations in which radical expressions include variables are known as **radical equations.** To solve radical equations, first isolate the radical on one side of the equation. Then raise each side of the equation to the proper power to eliminate the radical expression. This process of raising each side of an equation to a power often introduces **extraneous solutions.** Therefore, it is important to check all possible solutions in the original equation to determine if any of them should be eliminated from the solution set. **Radical inequalities** are solved using the same techniques used for solving radical equations.

Example 1 **Solve $3 = \sqrt[3]{x^2 - 2x + 1} - 1$.**

$3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$4 = \sqrt[3]{x^2 - 2x + 1}$ *Isolate the cube root.*

$64 = x^2 - 2x + 1$ *Cube each side.*

$0 = x^2 - 2x - 63$

$0 = (x - 9)(x + 7)$ *Factor.*

$x - 9 = 0$ $\quad$ $x + 7 = 0$

$x = 9$ $\quad$ $x = -7$

Check both solutions to make sure they are not extraneous.

$x = 9$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(9)^2 - 2(9) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

$x = -7$: $3 = \sqrt[3]{x^2 - 2x + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{(-7)^2 - 2(-7) + 1} - 1$

$3 \stackrel{?}{=} \sqrt[3]{64} - 1$

$3 \stackrel{?}{=} 4 - 1$

$3 = 3$ ✔

Example 2 **Solve $2\sqrt{3x + 5} > 2$.**

$2\sqrt{3x + 5} > 2$

$4(3x + 5) > 4$ *Square each side.*

$3x + 5 > 1$ *Divide each side by 4.*

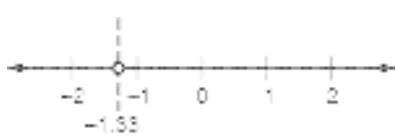
$3x > -4$

$x > -1.33$

In order for $\sqrt{3x + 5}$ to be a real number, $3x + 5$ must be greater than or equal to zero.

$3x + 5 \geq 0$

$3x \geq -5$

$x \geq -1.67$

Since -1.33 is greater than -1.67, the solution is $x > -1.33$. Check this solution by testing values in the intervals defined by the solution. Then graph the solution on a number line.