4-1

Study Guide

Polynomial Functions

The **degree** of a polynomial in one variable is the greatest exponent of its variable. The coefficient of the variable with the greatest exponent is called the **leading coefficient.** If a function $f(x)$ is defined by a polynomial in one variable, then it is a polynomial function. The values of x for which $f(x) = 0$ are called the **zeros** of the function. Zeros of the function are **roots** of the **polynomial equation** when $f(x) = 0$. A polynomial equation of degree n has exactly n roots in the set of complex numbers.

Example 1 **State the degree and leading coefficient of the polynomial function $f(x) = 6x^5 + 8x^3 - 8x$. Then determine whether $\sqrt{\frac{2}{3}}$ is a zero of $f(x)$.**

$6x^5 + 8x^3 - 8x$ has a degree of 5 and a leading coefficient of 6.

Evaluate the function for $x = \sqrt{\frac{2}{3}}$. That is, find $f\left(\sqrt{\frac{2}{3}}\right)$.

$$f\left(\sqrt{\frac{2}{3}}\right) = 6\left(\sqrt{\frac{2}{3}}\right)^5 + 8\left(\sqrt{\frac{2}{3}}\right)^3 - 8\left(\sqrt{\frac{2}{3}}\right) \qquad x = \sqrt{\frac{2}{3}}$$

$$= \frac{24}{9}\sqrt{\frac{2}{3}} + \frac{16}{3}\sqrt{\frac{2}{3}} - 8\sqrt{\frac{2}{3}}$$

$$= 0$$

Since $f\left(\sqrt{\frac{2}{3}}\right) = 0$, $\sqrt{\frac{2}{3}}$ is a zero of $f(x) = 6x^5 + 8x^3 - 8x$.

Example 2 **Write a polynomial equation of least degree with roots 0, $\sqrt{2}i$, and $-\sqrt{2}i$.**

The linear factors for the polynomial are $x - 0$, $x - \sqrt{2}i$, and $x + \sqrt{2}i$. Find the products of these factors.

$$(x - 0)(x - \sqrt{2}i)(x + \sqrt{2}i) = 0$$
$$x(x^2 - 2i^2) = 0$$
$$x(x^2 + 2) = 0 \qquad -2i^2 = -2(-1) \text{ or } 2$$
$$x^3 + 2x = 0$$

Example 3 **State the number of complex roots of the equation $3x^2 + 11x - 4 = 0$. Then find the roots.**

The polynomial has a degree of 2, so there are two complex roots. Factor the equation to find the roots.

$$3x^2 + 11x - 4 = 0$$
$$(3x - 1)(x + 4) = 0$$

To find each root, set each factor equal to zero.

$$3x - 1 = 0 \qquad x + 4 = 0$$
$$3x = 1 \qquad x = -4$$
$$x = \frac{1}{3}$$

The roots are -4 and $\frac{1}{3}$.