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Study Guide

Mathematical Induction

A method of proof called **mathematical induction** can be used to prove certain conjectures and formulas. The following example demonstrates the steps used in proving a summation formula by mathematical induction.

Example **Prove that the sum of the first n positive even integers is $n(n + 1)$.**

Here S_n is defined as $2 + 4 + 6 + \cdots + 2n = n(n + 1)$.

1. First, verify that S_n is valid for the first possible case, $n = 1$. Since the first positive even integer is 2 and $1(1 + 1) = 2$, the formula is valid for $n = 1$.

2. Then, assume that S_n is valid for $n = k$.

$S_k \Rightarrow 2 + 4 + 6 + \cdots + 2k = k(k + 1)$. *Replace n with k.*

Next, prove that S_n is also valid for $n = k + 1$.

$S_{k+1} \Rightarrow 2 + 4 + 6 + \cdots + 2k + 2(k + 1)$

$= k(k + 1) + 2(k + 1)$ *Add 2(k + 1) to both sides.*

We can simplify the right side by adding $k(k + 1) + 2(k + 1)$.

$S_{k+1} \Rightarrow 2 + 4 + 6 + \cdots + 2k + 2(k + 1)$

$= (k + 1)(k + 2)$ *(k + 1) is a common factor.*

If $k + 1$ is substituted into the original formula ($n(n + 1)$), the same result is obtained.

$(k + 1)[(k + 1) + 1]$ or $(k + 1)(k + 2)$

Thus, if the formula is valid for $n = k$, it is also valid for $n = k + 1$. Since S_n is valid for $n = 1$, it is also valid for $n = 2$, $n = 3$, and so on. That is, the formula for the sum of the first n positive even integers holds.