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Study Guide

Products and Quotients of Complex Numbers in Polar Form

Example 1 **Find the product $2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) \cdot 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$. Then express the product in rectangular form.**

Find the modulus and amplitude of the product.

$$\begin{aligned} r &= r_1 r_2 \\ &= 2(4) \\ &= 8 \end{aligned} \qquad \begin{aligned} \theta &= \theta_1 + \theta_2 \\ &= \frac{\pi}{2} + \frac{\pi}{3} \\ &= \frac{5\pi}{6} \end{aligned}$$

The product is $8\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right)$.

Now find the rectangular form of the product.

$$\begin{aligned} 8\left(\cos \frac{5\pi}{6} + \boldsymbol{i} \sin \frac{5\pi}{6}\right) &= 8\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\boldsymbol{i}\right) \qquad \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2},\ \sin \frac{5\pi}{6} = \frac{1}{2} \\ &= -4\sqrt{3} + 4\boldsymbol{i} \end{aligned}$$

The rectangular form of the product is $-4\sqrt{3} + 4\boldsymbol{i}$.

Example 2 **Find the quotient $21\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) \div 7\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$. Then express the quotient in rectangular form.**

Find the modulus and amplitude of the quotient.

$$\begin{aligned} r &= \frac{r_1}{r_2} \\ &= \frac{21}{7} \\ &= 3 \end{aligned} \qquad \begin{aligned} \theta &= \theta_1 - \theta_2 \\ &= \frac{7\pi}{6} - \frac{4\pi}{3} \\ &= -\frac{\pi}{6} \end{aligned}$$

The quotient is $3\left[\cos\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{6}\right)\right]$.

Now find the rectangular form of the quotient.

$$\begin{aligned} 3\left[\cos\left(-\frac{\pi}{6}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{6}\right)\right] &= 3\left[\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\boldsymbol{i}\right] \qquad \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2},\ \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2} \\ &= \frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i} \end{aligned}$$

The rectangular form of the quotient is $\frac{3\sqrt{3}}{2} - \frac{3}{2}\boldsymbol{i}$.