13-5

Study Guide

Conditional Probabilities

The **conditional probability** of event A, given event B, is defined as $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$, where $P(B) \neq 0$. In some situations, event A is a subset of event B. In these situations, $P(A \mid B) = \frac{P(A)}{P(B)}$, where $P(B) \neq 0$.

Example **Each of four boxes contains a red marble and a yellow marble. A marble is selected from each box without looking. What is the probability that exactly three red marbles are selected if the third marble is red?**

Sample spaces and reduced sample spaces can be used to help determine the outcomes that satisfy a given condition.

The sample space is $S = \{RRRR, RRRY, RRYR, RRYY, RYRR, RYRY, RYYR, RYYY, YRRR, YRRY, YRYR, YRYY, YYRR, YYRY, YYYR, YYYY\}$ and includes all of the possible outcomes of selecting 1 of the marbles from each of the 4 boxes. All of the outcomes are equally likely.

Event B represents the condition that the third marble is red.
$B = \{RRRR, RRRY, RYRR, RYRY, YRRR, YRRY, YYRR, YYRY\}$

$P(B) = \frac{8}{16}$ or $\frac{1}{2}$

Event A represents the condition that exactly three of the marbles are red.

$A = \{RRRY, RRYR, RYRR, YRRR\}$

(A and B) is the intersection of A and B.
(A and B) $= \{RRRY, RYRR, YRRR\}$.

So, $P(A \text{ and } B) = \frac{3}{16}$.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{\frac{3}{16}}{\frac{1}{2}} \text{ or } \frac{3}{8}$$

The probability that exactly three marbles are red given that the third marble is red is $\frac{3}{8}$.