12-3

Study Guide

Infinite Sequences and Series

An **infinite sequence** is one that has infinitely many terms. An **infinite series** is the indicated sum of the terms of an infinite sequence.

Example 1 **Find $\lim\limits_{n\to\infty} \frac{4n^2 - n + 3}{n^2 + 1}$.**

Divide each term in the numerator and the denominator by the highest power of n to produce an equivalent expression. In this case, n^2 is the highest power.

$$\lim \frac{4n^2 - n + 3}{n^2 + 1} = \lim \frac{\frac{4n^2}{n^2} - \frac{n}{n^2} + \frac{3}{n^2}}{\frac{n^2}{n^2} + \frac{1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{4 - \frac{1}{n} + \frac{3}{n^2}}{1 + \frac{1}{n^2}} \qquad \textit{Simplify.}$$

$$= \frac{\lim\limits_{n\to\infty} 4 - \lim\limits_{n\to\infty} \frac{1}{n} + \lim\limits_{n\to\infty} 3 \cdot \lim\limits_{n\to\infty} \frac{1}{n^2}}{\lim\limits_{n\to\infty} 1 + \lim\limits_{n\to\infty} \frac{1}{n^2}} \qquad \textit{Apply limit theorems.}$$

$$= \frac{4 - 0 + 3 \cdot 0}{1 + 0} \text{ or } 4 \qquad \lim_{n\to\infty} 4 = 4,\ \lim_{n\to\infty} \frac{1}{n} = 0,\ \lim_{n\to\infty} 3 = 3,\ \lim_{n\to\infty} \frac{1}{n^2} = 0,\ \lim_{n\to\infty} 1 = 1$$

Thus, the limit is 4.

Example 2 **Find the sum of the series $\frac{3}{2} - \frac{3}{8} + \frac{3}{32} - \cdots$.**

In the series $a_1 = \frac{3}{2}$ and $r = -\frac{1}{4}$.

Since $|r| < 1$, $S = \frac{a_1}{1 - r}$.

$$S = \frac{a_1}{1 - r} = \frac{\frac{3}{2}}{1 - \left(-\frac{1}{4}\right)} \qquad a_1 = \frac{3}{2} \textit{ and } r = -\frac{1}{4}$$

$$= \frac{12}{10} \text{ or } 1\frac{1}{5}$$

The sum of the series is $1\frac{1}{5}$.