10-6

Study Guide

Rectangular and Parametric Forms of Conic Sections

Use the table to identify a conic section given its equation in general form.

conic	$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
circle	$A = C$
parabola	Either A or C is zero.
ellipse	A and C have the same sign and $A \neq C$.
hyperbola	A and C have opposite signs.

Example 1 **Identify the conic section represented by the equation $5x^2 + 4y^2 - 10x - 8y + 18 = 0$.**

$A = 5$ and $C = 4$. Since A and C have the same signs and are not equal, the conic is an ellipse.

Example 2 **Find the rectangular equation of the curve whose parametric equations are $x = 2t$ and $y = 4t^2 + 4t - 1$. Then identify the conic section represented by the equation.**

First, solve the equation $x = 2t$ for t.

$x = 2t$

$\frac{x}{2} = t$

Then substitute $\frac{x}{2}$ for t in the equation $y = 4t^2 + 4t - 1$.

$y = 4t^2 + 4t - 1$

$y = 4\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right) - 1 \qquad t = \frac{x}{2}$

$y = x^2 + 2x - 1$

Since $C = 0$, the equation $y = x^2 + 2x - 1$ is the equation of a parabola.

Example 3 **Find the rectangular equation of the curve whose parametric equations are $x = 3 \cos t$ and $y = 5 \sin t$, where $0 \leq t \leq 2\pi$. Then graph the equation using arrows to indicate orientation.**

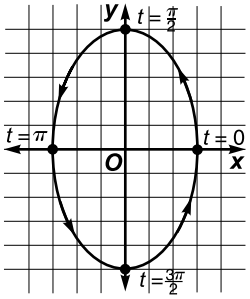
Solve the first equation for $\cos t$ and the second equation for $\sin t$.

$\cos t = \frac{x}{3}$ and $\sin t = \frac{y}{5}$

Use the trigonometric identity $\cos^2 t + \sin^2 t = 1$ to eliminate t.

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1 \qquad$ *Substitution*

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

This is the equation of an ellipse with the center at (0, 0). As t increases from 0 to 2π, the curve is traced in a counterclockwise motion.