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Study Guide

Vectors in Three-Dimensional Space

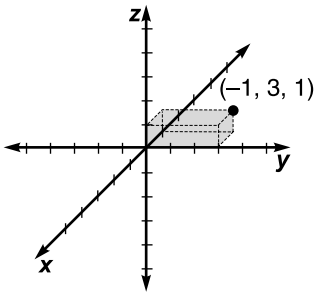
Ordered triples, like ordered pairs, can be used to represent vectors. Operations on vectors respresented by ordered triples are similar to those on vectors represented by ordered pairs. For example, an extension of the formula for the distance between two points in a plane allows us to find the distance between two points in space.

Example 1 **Locate the point at (−1, 3, 1).**

Locate −1 on the x-axis, 3 on the y-axis, and 1 on the z-axis.

Now draw broken lines for parallelograms to represent the three planes.

The planes intersect at (−1, 3, 1).



Example 2 **Write the ordered triple that represents the vector from $X(-4, 5, 6)$ to $Y(-2, 6, 3)$. Then find the magnitude of $\overrightarrow{XY}$.**

$$\begin{aligned}\overrightarrow{XY} &= (-2, 6, 3) - (-4, 5, 6)\\ &= \langle -2 - (-4), 6 - 5, 3 - 6\rangle\\ &= \langle 2, 1, -3\rangle\end{aligned}$$

$$\begin{aligned}|\overrightarrow{XY}| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\\ &= \sqrt{[-2 - (-4)]^2 + (6 - 5)^2 + (3 - 6)^2}\\ &= \sqrt{(2)^2 + (1)^2 + (-3)^2}\\ &= \sqrt{14} \text{ or } 3.7\end{aligned}$$

Example 3 **Find an ordered triple that represents $2\vec{s} + 3\vec{t}$ if $\vec{s} = \langle 5, -1, 2\rangle$ and $\vec{t} = \langle 4, 3, -2\rangle$.**

$$\begin{aligned}2\vec{s} + 3\vec{t} &= 2\langle 5, -1, 2\rangle + 3\langle 4, 3, -2\rangle\\ &= \langle 10, -2, 4\rangle + \langle 12, 9, -6\rangle\\ &= \langle 22, 7, -2\rangle\end{aligned}$$

Example 4 **Write $\overrightarrow{AB}$ as the sum of unit vectors for $A(5, -2, 3)$ and $B(-4, 2, 1)$.**

First express $\overrightarrow{AB}$ as an ordered triple. Then write the sum of the unit vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$.

$$\begin{aligned}\overrightarrow{AB} &= (-4, 2, 1) - (5, -2, 3)\\ &= \langle -4 - 5, 2 - (-2), 1 - 3\rangle\\ &= \langle -9, 4, -2\rangle\\ &= -9\vec{i} + 4\vec{j} - 2\vec{k}\end{aligned}$$