12-1

Study Guide

Arithmetic Sequences and Series

A **sequence** is a function whose domain is the set of natural numbers. The **terms** of a sequence are the range elements of the function. The difference between successive terms of an **arithmetic sequence** is a constant called the **common difference**, denoted as d. An **arithmetic series** is the indicated sum of the terms of an arithmetic sequence.

Example 1 **a. Find the next four terms in the arithmetic sequence −7, −5, −3, ...**
b. Find the 38th term of this sequence.

a. Find the common difference.
$a_2 - a_1 = -5 - (-7)$ or 2

The common difference is 2. Add 2 to the third term to get the fourth term, and so on.
$a_4 = -3 + 2$ or -1 $\quad a_5 = -1 + 2$ or 1
$a_6 = 1 + 2$ or 3 $\quad a_7 = 3 + 2$ or 5

The next four terms are −1, 1, 3, and 5.

b. Use the formula for the nth term of an arithmetic sequence.
$a_n = a_1 + (n - 1)d$
$a_{38} = -7 + (38 - 1)2 \quad n = 38, a_1 = -7, d = 2$
$a_{38} = 67$

Example 2 **Write an arithmetic sequence that has three arithmetic means between 3.2 and 4.4.**

The sequence will have the form 3.2, ___?___, ___?___, ___?___, 4.4.

First, find the common difference.
$a_n = a_1 + (n - 1)d$
$4.4 = 3.2 + (5 - 1)d \quad n = 5, a_5 = 4.4, a_1 = 3.2$
$4.4 = 3.2 + 4d$
$d = 0.3$

Then, determine the arithmetic means.

a_2	a_3	a_4
3.2 + 0.3 = 3.5	3.5 + 0.3 = 3.8	3.8 + 0.3 = 4.1

The sequence is 3.2, 3.5, 3.8, 4.1, 4.4.

Example 3 **Find the sum of the first 50 terms in the series 11 + 14 + 17 + · · · + 158.**

$S_n = \frac{n}{2}(a_1 + a_n)$
$S_{50} = \frac{50}{2}(11 + 158) \quad n = 50, a_1 = 11, a_{50} = 158$
$= 4225$