10-7

Study Guide

Transformation of Conics

Translations are often written in the form $T_{(h, k)}$. To find the equation of a rotated conic, replace x with $x' \cos \theta + y' \sin \theta$ and y with $-x' \sin \theta + y' \cos \theta$.

Example **Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.**

a. $\boldsymbol{4x^2 + y^2 = 12}$ **for** $\boldsymbol{T_{(-2, 3)}}$

The graph of this equation is an ellipse. To write the equation of $4x^2 + y^2 = 12$ for $T_{(-2, 3)}$, let $h = -2$ and $k = 3$. Then replace x with $x - h$ and y with $y - k$.

$x^2 \Rightarrow (x - (-2))^2$ or $(x + 2)^2$
$y^2 \Rightarrow (y - 3)^2$

Thus, the translated equation is $4(x + 2)^2 + (y - 3)^2 = 12$.

Write the equation in general form.

$$\begin{aligned} 4(x + 2)^2 + (y - 3)^2 &= 12 \\ 4(x^2 + 4x + 4) + y^2 - 6y + 9 &= 12 && \textit{Expand the binomial.} \\ 4x^2 + y^2 + 16x - 6y + 25 &= 12 && \textit{Simplify.} \\ 4x^2 + y^2 + 16x - 6y + 13 &= 0 && \textit{Subtract 12 from both sides.} \end{aligned}$$

b. $\boldsymbol{x^2 - 4y = 0, \theta = 45°}$

The graph of this equation is a parabola. Find the expressions to replace x and y.

Replace x with $x' \cos 45° + y' \sin 45°$ or $\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$.

Replace y with $-x' \sin 45° + y' \cos 45°$ or $-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$.

$$\begin{aligned} x^2 - 4y &= 0 \\ \left(\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right)^2 - 4\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) &= 0 && \textit{Replace } x \textit{ and } y. \\ \left[\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2\right] - 4\left(-\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'\right) &= 0 && \textit{Expand the binomial.} \\ \frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 + 2\sqrt{2}x' - 2\sqrt{2}y' &= 0 && \textit{Simplify.} \end{aligned}$$

The equation of the parabola after the 45° rotation is

$\frac{1}{2}(x')^2 + x'y' + \frac{1}{2}(y')^2 + 2\sqrt{2}x' - 2\sqrt{2}y' = 0$