12-7

Study Guide

Special Sequences and Series

The value of e^x can be approximated by using the **exponential series.** The **trigonometric series** can be used to approximate values of the trigonometric functions. **Euler's formula** can be used to write the exponential form of a complex number and to find a complex number that is the natural logarithm of a negative number.

Example 1 **Use the first five terms of the trigonometric series to approximate the value of $\sin \frac{\pi}{6}$ to four decimal places.**

$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

Let $x = \frac{\pi}{6}$, or about 0.5236.

$\sin \frac{\pi}{6} \approx 0.5236 - \frac{(0.5236)^3}{3!} + \frac{(0.5236)^5}{5!} - \frac{(0.5236)^7}{7!} + \frac{(0.5236)^9}{9!}$

$\sin \frac{\pi}{6} \approx 0.5236 - 0.02392 + 0.00033 - 0.000002 + 0.000000008$

$\sin \frac{\pi}{6} \approx 0.5000$ *Compare this result to the actual value, 0.5.*

Example 2 **Write $4 - 4i$ in exponential form.**

Write the polar form of $4 - 4\boldsymbol{i}$.
Recall that $a + b\boldsymbol{i} = r(\cos \theta + \boldsymbol{i} \sin \theta)$, where
$r = \sqrt{a^2 + b^2}$ and $\theta = \text{Arctan } \frac{b}{a}$ when $a > 0$.

$r = \sqrt{4^2 + (-4)^2}$ or $4\sqrt{2}$, and *$a = 4$ and $b = -4$*

$\theta = \text{Arctan } \frac{-4}{4}$ or $-\frac{\pi}{4}$

$$4 - 4\boldsymbol{i} = 4\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + \boldsymbol{i} \sin\left(-\frac{\pi}{4}\right)\right]$$
$$= 4\sqrt{2}e^{-\boldsymbol{i}\frac{\pi}{4}}$$

Thus, the exponential form of $4 - 4\boldsymbol{i}$ is $4\sqrt{2}e^{-\boldsymbol{i}\frac{\pi}{4}}$.

Example 3 **Evaluate $\ln(-12.4)$.**

$\ln(-12.4) = \ln(-1) + \ln(12.4)$
$\approx \boldsymbol{i}\pi + 2.5177$ *Use a calculator to compute ln(12.4).*

Thus, $\ln(-12.4) \approx \boldsymbol{i}\pi + 2.5177$. *The logarithm is a complex number.*