13-2

Study Guide

Permutations with Repetitions and Circular Permutations

For permutations involving repetitions, the number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$. When n objects are arranged in a circle, there are $\frac{n!}{n}$, or $(n - 1)!$, permutations of the objects around the circle. If n objects are arranged relative to a fixed point, then there are $n!$ permutations.

Example 1 **How many 10-letter patterns can be formed from the letters of the word *basketball*?**

The ten letters can be arranged in $P(10, 10)$, or 10!, ways. However, some of these 3,628,800 ways have the same appearance because some of the letters appear more than once.

$\frac{10!}{2!2!2!}$ *There are 2 as, 2 bs, and 2 ls in basketball.*

$$\frac{10!}{2!2!2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$$

$$= 453{,}600$$

There are 453,600 ten-letter patterns that can be formed from the letters of the word *basketball*.

Example 2 **Six people are seated at a round table to play a game of cards.**

a. Is the seating arrangement around the table a linear or circular permutation? Explain.

b. How many possible seating arrangements are there?

a. The arrangement of people is a circular permutation since the people form a circle around the table.

b. There are 6 people, so the number of arrangements can be described by $(6 - 1)!$.

$$(6 - 1)! = 5!$$
$$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 120$$

There are 120 possible seating arrangements.