5-8

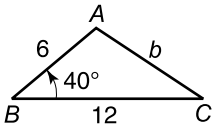
Study Guide

The Law of Cosines

When we know the measures of two sides of a triangle and the included angle, we can use the **Law of Cosines** to find the measure of the third side. Often times we will use both the Law of Cosines and the Law of Sines to solve a triangle.

Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **Solve $\triangle ABC$ if $B = 40°$, $a = 12$, and $c = 6$.**



$b^2 = a^2 + c^2 - 2ac \cos B$ *Law of Cosines*

$b^2 = 12^2 + 6^2 - 2(12)(6) \cos 40°$

$b^2 \approx 69.68960019$

$b \approx 8.348029719$

So, $b \approx 8.3$.

$\frac{b}{\sin B} = \frac{c}{\sin C}$ *Law of Sines*

$\frac{8.3}{\sin 40°} = \frac{6}{\sin C}$

$\sin C = \frac{6 \sin 40°}{8.3}$

$C = \sin^{-1}\left(\frac{6 \sin 40°}{8.3}\right)$

$C \approx 27.68859159$

So, $C \approx 27.7°$.

$A \approx 180° - (40° + 27.7°) \approx 112.3°$

The solution of this triangle is $b \approx 8.3$, $A \approx 112.3°$, and $C \approx 27.7°$.

Example 2 **Find the area of $\triangle ABC$ if $a = 5$, $b = 8$, and $c = 10$.**

First, find the semiperimeter of $\triangle ABC$.

$s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(5 + 8 + 10)$

$s = 11.5$

Now, apply Hero's Formula

$k = \sqrt{s(s - a)(s - b)(s - c)}$

$k = \sqrt{11.5(11.5 - 5)(11.5 - 8)(11.5 - 10)}$

$k = \sqrt{392.4375}$

$k \approx 19.81003534$

The area of the triangle is about 19.8 square units.