NAME _____ DATE _____ PERIOD _____

Study Guide

Polar and Rectangular Coordinates

Use the conversion formulas in the following examples to convert coordinates and equations from one coordinate system to the other.

Example 1 **Find the rectangular coordinates of each point.**

a. $P\left(3, \frac{3\pi}{4}\right)$

For $P\left(3, \frac{3\pi}{4}\right)$, $r = 3$ and $\theta = \frac{3\pi}{4}$.

Use the conversion formulas $x = r\cos\theta$ and $y = r\sin\theta$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 3\cos\frac{3\pi}{4} & &= 3\sin\frac{3\pi}{4} \\ &= 3\left(-\frac{\sqrt{2}}{2}\right) & &= 3\left(\frac{\sqrt{2}}{2}\right) \\ &\text{or } -\frac{3\sqrt{2}}{2} & &\text{or } \frac{3\sqrt{2}}{2} \end{aligned}$$

The rectangular coordinates of P are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, or $(-2.12, 2.12)$ to the nearest hundredth.

b. $Q(20, -60°)$

For $Q(20, -60°)$, $r = 20$ and $\theta = -60°$.

$$\begin{aligned} x &= r\cos\theta & y &= r\sin\theta \\ &= 20\cos(-60°) & &= 20\sin(-60°) \\ &= 20(0.5) & &= 20\left(-\frac{\sqrt{3}}{2}\right) \\ &= 10 & &= -10\sqrt{3} \end{aligned}$$

The rectangular coordinates of Q are $(10, -10\sqrt{3})$, or approximately $(10, -17.32)$

Example 2 **Find the polar coordinates of $R(5, -9)$.**

For $R(5, -9)$, $x = 5$ and $y = -9$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{5^2 + (-9)^2} \\ &= \sqrt{106} \text{ or about } 10.30 \end{aligned}$$

$$\begin{aligned} \theta &= \text{Arctan}\,\frac{y}{x} \qquad x > 0 \\ &= \text{Arctan}\,\frac{-9}{5} \\ &\approx -1.06 \end{aligned}$$

To obtain an angle between 0 and 2π you can add 2π to the θ-value. This results in $\theta = 5.22$.

The polar coordinates of R are approximately $(10.30, 5.22)$.

Example 3 **Write the polar equation $r = 5\cos\theta$ in rectangular form.**

$$\begin{aligned} r &= 5\cos\theta \\ r^2 &= 5r\cos\theta && \textit{Multiply each side by } r. \\ x^2 + y^2 &= 5x && r^2 = x^2 + y^2 \textit{ and } r\cos\theta = x \end{aligned}$$