11-2

Study Guide

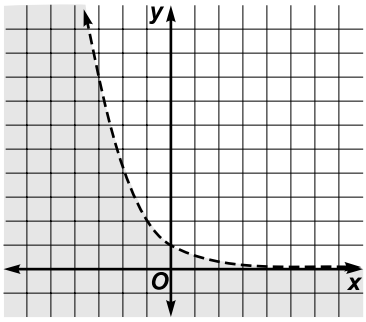
Exponential Functions

Functions of the form $y = b^x$, in which the base b is a positive real number and the exponent is a variable, are known as **exponential functions.** Many real-world situations can be modeled by exponential functions. The equation $N = N_0(1 + r)^t$, where N is the final amount, N_0 is the initial amount, r is the rate of growth or decay, and t is time, is used for modeling exponential growth. The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is compounded each year, and t is the number of years.

Example 1 **Graph $y < 2^{-x}$.**

First, graph $y = 2^{-x}$. This graph is a function, since there is a unique y-value for each x-value.

x	−3	−2	−1	0	1	2	3	4
2^{-x}	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$



Since the points on this curve are not in the solution of the inequality, the graph of $y = 2^{-x}$ is shown as a dashed curve.

Then, use (0, 0) as a test point to determine which area to shade.

$y < 2^{-x}$

$0 < 2^0$

$0 < 1$

Since (0,0) satisfies the inequality, the region that contains (0,0) should be shaded.

Example 2 ***Biology*** **Suppose a researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?**

$N = N_0(1 + r)^t$

$N = 100(1 + 0.25)^6$ $\quad N_0 = 100, r = 0.25, t = 6$

$N \approx 381.4697266$ $\quad$ *Use a calculator.*

There will be about 381 cells in the colony in 6 weeks.

Example 3 ***Finance*** **Determine the amount of money in a money market account that provides an annual rate of 6.3% compounded quarterly if $1700 is invested and left in the account for eight years.**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 1700\left(1 + \frac{0.063}{4}\right)^{4 \cdot 8}$ $\quad P = 1700, r = 0.063, n = 4, t = 8$

$A \approx 2803.028499$ $\quad$ *Use a calculator.*

After 8 years, the $1700 investment will have a value of $2803.03.