10-2

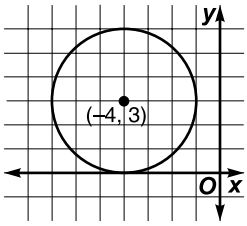
Study Guide

Circles

The standard form of the equation of a **circle** with **radius** r and **center** at (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1 **Write the standard form of the equation of the circle that is tangent to the x-axis and has its center at $(-4, 3)$. Then graph the equation.**

Since the circle is tangent to the x-axis, the distance from the center to the x-axis is the radius. The center is 3 units above the x-axis. Therefore, the radius is 3.



$(x - h)^2 + (y - k)^2 = r^2$ *Standard form*

$[x - (-4)]^2 + (y - 3)^2 = 3^2$ *$(h, k) = (-4, 3)$ and $r = 3$*

$(x + 4)^2 + (y - 3)^2 = 9$

Example 2 **Write the standard form of the equation of the circle that passes through the points at $(1, -1)$, $(5, 3)$, and $(-3, 3)$. Then identify the center and radius of the circle.**

Substitute each ordered pair (x, y) in the general form $x^2 + y^2 + Dx + Ey + F = 0$ to create a system of equations.

$(1)^2 + (-1)^2 + D(1) + E(-1) + F = 0$ *$(x, y) = (1, -1)$*

$(5)^2 + (3)^2 + D(5) + E(3) + F = 0$ *$(x, y) = (5, 3)$*

$(-3)^2 + (3)^2 + D(-3) + E(3) + F = 0$ *$(x, y) = (-3, 3)$*

Simplify the system of equations.

$D - E + F + 2 = 0$

$5D + 3E + F + 34 = 0$

$-3D + 3E + F + 18 = 0$

The solution to the system is $D = -2$, $E = -6$, and $F = -6$.

The general form of the equation of the circle is $x^2 + y^2 - 2x - 6y - 6 = 0$.

$x^2 + y^2 - 2x - 6y - 6 = 0$

$(x^2 - 2x + ?) + (y^2 - 6y + ?) = 6$ *Group to form perfect square trinomials.*

$(x^2 - 2x + 1) + (y^2 - 6y + 9) = 6 + 1 + 9$ *Complete the square.*

$(x - 1)^2 + (y - 3)^2 = 16$ *Factor the trinomials.*

After completing the square, the standard form of the circle is $(x - 1)^2 + (y - 3)^2 = 16$. Its center is at $(1, 3)$, and its radius is 4.