7-4

Study Guide

Double-Angle and Half-Angle Identities

Example 1 **If $\sin\theta = \frac{1}{4}$ and θ has its terminal side in the first quadrant, find the exact value of $\sin 2\theta$.**

To use the double-angle identity for $\sin 2\theta$, we must first find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1 \qquad \sin\theta = \frac{1}{4}$$

$$\cos^2\theta = \frac{15}{16}$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

Now find $\sin 2\theta$.

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \textit{Double-angle identity for sine}$$

$$= 2\left(\frac{1}{4}\right)\frac{\sqrt{15}}{4} \qquad \sin\theta = \frac{1}{4},\ \cos\theta = \frac{\sqrt{15}}{4}$$

$$= \frac{\sqrt{15}}{8}$$

Example 2 **Use a half-angle identity to find the exact value of $\sin\frac{\pi}{12}$.**

$$\sin\frac{\pi}{12} = \sin\frac{\frac{\pi}{6}}{2}$$

$$= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} \qquad \textit{Use } \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}. \textit{ Since } \frac{\pi}{12} \textit{ is in Quadrant I, choose the positive sine value.}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{2}$$