3-6

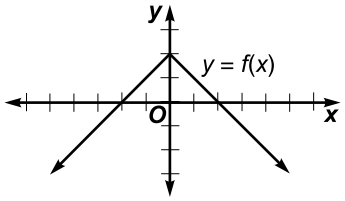
Study Guide

Critical Points and Extrema

Critical points are points on a graph at which a line drawn tangent to the curve is horizontal or vertical. A critical point may be a **maximum**, a **minimum**, or a **point of inflection.** A point of inflection is a point where the graph changes its curvature. Graphs can have an **absolute maximum**, an **absolute minimum**, a **relative maximum**, or a **relative minimum.** The general term for maximum or minimum is **extremum** (plural, *extrema*).

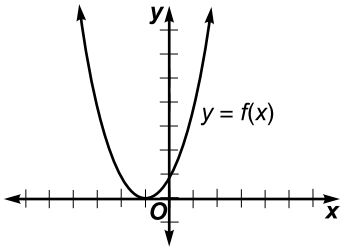
Example 1 **Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.**

a.



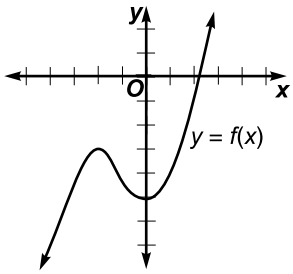
The function has an absolute maximum at (0, 2). The absolute maximum is the greatest value that a function assumes over its domain.

b.



The function has an absolute minimum at (−1, 0). The absolute minimum is the least value that a function assumes over its domain.

c.



The relative maximum and minimum may not be the greatest and the least y-value for the domain, respectively, but they are the greatest and least y-value on some interval of the domain. The function has a relative maximum at (−2, −3) and a relative minimum at (0, −5). Because the graph indicates that the function increases or decreases without bound as x increases or decreases, there is neither an absolute maximum nor an absolute minimum.

By testing points on both sides of a critical point, you can determine whether the critical point is a relative maximum, a relative minimum, or a point of inflection.

Example 2 **The function $f(x) = 2x^6 + 2x^4 - 9x^2$ has a critical point at $x = 0$. Determine whether the critical point is the location of a maximum, a minimum, or a point of inflection.**

x	$x - 0.1$	$x + 0.1$	$f(x - 0.1)$	$f(x)$	$f(x + 0.1)$	Type of Critical Point
0	−0.1	0.1	−0.0899	0	−0.0899	maximum

Because 0 is greater than both $f(x - 0.1)$ and $f(x + 0.1)$, $x = 0$ is the location of a relative maximum.