7-3

Study Guide

Sum and Difference Identities

You can use the **sum and difference identities** and the values of the trigonometric functions of common angles to find the values of trigonometric functions of other angles. Notice how the addition and subtraction symbols are related in the sum and difference identities.

Sum and Difference Identities	
Cosine function	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
Sine function	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
Tangent function	$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Example 1 **Use the sum or difference identity for cosine to find the exact value of cos 375°.**

$375° = 360° + 15°$

$\cos 375° = \cos 15°$ *Symmetry identity, Case 1*

$\cos 15° = \cos(60° - 45°)$ *60° and 45° are two common angles that differ by 15°.*

$\cos 15° = \cos 60° \cos 45° + \sin 60° \sin 45°$ *Difference identity for cosine*

$\cos 15° = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$ or $\frac{\sqrt{2} + \sqrt{6}}{4}$

Example 2 **Find the value of $\sin(x + y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $\sin x = \frac{3}{5}$, and $\sin y = \frac{12}{37}$.**

In order to use the sum identity for sine, you need to know $\cos x$ and $\cos y$. Use a Pythagorean identity to determine the necessary values.

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \sin^2\alpha.$ *Pythagorean identity*

Since it is given that the angles are in Quadrant I, the values of sine and cosine are positive. Therefore, $\cos\alpha = \sqrt{1 - \sin^2\alpha}$.

$\cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$ $\qquad$ $\cos y = \sqrt{1 - \left(\frac{12}{37}\right)^2}$

$= \sqrt{\frac{16}{25}}$ or $\frac{4}{5}$ $\qquad$ $= \sqrt{\frac{1225}{1369}}$ or $\frac{35}{37}$

Now substitute these values into the sum identity for sine.

$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{3}{5}\right)\left(\frac{35}{37}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{37}\right)$ or $\frac{153}{185}$