10-8

Study Guide

Systems of Second-Degree Equations and Inequalities

To find the exact solution to a system of second-degree equations, you must use algebra. Graph systems of inequalities involving second-degree equations to find solutions for the inequality.

Example **a. Solve the system of equations algebraically. Round to the nearest tenth.**

$\boldsymbol{x^2 + 2y^2 = 9}$
$\boldsymbol{3x^2 - y^2 = 1}$

Since both equations contain a single term involving y, you can solve the system as follows.

First, multiply each side of the second equation by 2.

$$2(3x^2 - y^2) = 2(1)$$
$$6x^2 - 2y^2 = 2$$

Then, add the equations.

$$\begin{aligned} 6x^2 - 2y^2 &= 2 \\ x^2 + 2y^2 &= 9 \\ 7x^2 \qquad &= 11 \\ x &= \pm\sqrt{\frac{11}{7}} \end{aligned}$$

Now find y by substituting $\pm\sqrt{\frac{11}{7}}$ for x in one of the original equations.

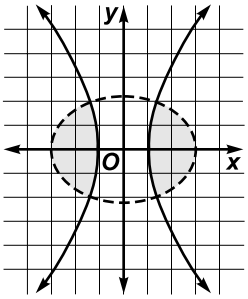
$$\begin{aligned} x^2 + 2y^2 = 9 \rightarrow \left(\pm\sqrt{\frac{11}{7}}\right)^2 + 2y^2 &= 9 \\ 11 + 14y^2 &= 63 \\ y^2 &= \frac{26}{7} \\ y &\approx \pm 1.9 \end{aligned}$$

The solutions are $(1.3, \pm 1.9)$, and $(-1.3, \pm 1.9)$.

b. Graph the solutions for the system of inequalities.

$\boldsymbol{x^2 + 2y^2 < 9}$
$\boldsymbol{3x^2 - y^2 \geq 1}$

First graph $x^2 + 2y^2 < 9$. The ellipse should be dashed. Test a point either inside or outside the ellipse to see if its coordinates satisfy the inequality. Since (0, 0) satisfies the inequality, shade the interior of the ellipse.



Then graph $3x^2 - y^2 \geq 1$. The hyperbola should be a solid curve. Test a point inside the branches of the hyperbola or outside its branches. Since (0, 0) does not satisfy the inequality, shade the regions inside the branches. The intersection of the two graphs represents the solution of the system.