15-4

Study Guide

The Fundamental Theorem of Calculus

The **Fundamental Theorem of Calculus** states that if $F(x)$ is the antiderivative of the continuous function $f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

The statement may also be written $\int_a^b f(x)\,dx = F(x)\Big|_a^b$.

The antiderivative rules for **indefinite integrals,** denoted $\int f(x)\,dx$, are as follows.

Power Rule:	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, where n is a rational number and $n \neq -1$.
Constant Multiple of a Power Rule:	$\int kx^n\,dx = k \cdot \frac{1}{n+1}x^{n+1} + C$, where k is a constant, n is a rational number, and $n \neq -1$.
Sum and Difference Rule:	$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Example 1 **Evaluate $\int_2^5 x^2\,dx$.**

The antiderivative of $f(x) = x^2$ is $F(x) = \frac{1}{3}x^3 + C$

$$\int_2^5 x^2\,dx = \frac{1}{3}x^3 + C\Big|_2^5 \qquad \textit{Fundamental Theorem of Calculus}$$

$$= \left(\frac{1}{3} \cdot 5^3 + C\right) - \left(\frac{1}{3} \cdot 2^3 + C\right) \qquad \textit{Let } x = 5 \textit{ and } 2 \textit{ and subtract.}$$

$$= \frac{117}{3} \text{ or } 39$$

Notice that the constant term C is eliminated when evaluating a definite integral using the Fundamental Theorem of Calculus.

Example 2 **Evaluate the indefinite integral $\int (3x^2 + 4x - 1)\,dx$.**

$$\int (3x^2 + 4x - 1)\,dx = \int (3x^2 + 4x^1 - x^0)\,dx \qquad x = x^1,\ 1 = x^0$$

$$= 3 \cdot \frac{1}{2+1}x^{2+1} + 4 \cdot \frac{1}{1+1}x^{1+1} - \left(\frac{1}{0+1}x^{0+1}\right) + C$$

Use antiderivative rules.

$$= \frac{3}{3}x^3 + \frac{4}{2}x^2 - \frac{1}{1}x^1 + C$$

$$= x^3 + 2x^2 - x + C \qquad \textit{Simplify.}$$