12-5

Study Guide

Sigma Notation and the *n*th Term

A series may be written using **sigma notation.**

maximum value of n → $\sum_{n=1}^{k} a_n$ ← *expression for general term*

starting value of n → $n=1$ ↑ *index of summation*

Example 1 **Write each expression in expanded form and then find the sum.**

a. $\sum_{n=1}^{5} (n + 2)$

First, write the expression in expanded form.

$$\sum_{n=1}^{5} (n + 2) = (1 + 2) + (2 + 2) + (3 + 2) + (4 + 2) + (5 + 2)$$

Then, find the sum by simplifying the expanded form. $3 + 4 + 5 + 6 + 7 = 25$

b. $\sum_{m=1}^{\infty} 2\left(\frac{1}{4}\right)^m$

$$\sum_{m=1}^{\infty} 2\left(\frac{1}{4}\right)^m = 2\left(\frac{1}{4}\right)^1 + 2\left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right)^3 + \cdots$$

$$= \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$$

This is an infinite series. Use the formula $S = \frac{a_1}{1 - r}$.

$S = \frac{\frac{1}{2}}{1 - \frac{1}{4}}$ $\quad a_1 = \frac{1}{2}, r = \frac{1}{4}$

$S = \frac{2}{3}$

Example 2 **Express the series 26 + 37 + 50 + 65 + · · · + 170 using sigma notation.**

Notice that each term is one more than a perfect square. Thus, the *n*th term of the series is $n^2 + 1$. Since $5^2 + 1 = 26$ and $13^2 + 1 = 170$, the index of summation goes from $n = 5$ to $n = 13$.

Therefore, $26 + 37 + 50 + 65 + \cdots + 170 = \sum_{n=5}^{13} (n^2 + 1)$.