15-1

Study Guide

Limits

If the function $f(x)$ is continuous at a, then $\lim_{x \to a} f(x) = f(a)$. Continuous functions include polynomials as well as the functions $\sin x$, $\cos x$, and a^x. Also, $\log_a x$ is continuous if $x > 0$. To evaluate the **limit** when a function is not continuous at the x-value in question, apply algebraic methods to decompose the function into a simpler one. If the function cannot be simplified, compute values of the function using x-values that get closer and closer to a from either side.

Example 1 **Evaluate each limit.**

a. $\lim_{x \to 1} (x^4 - 2x^2 + x + 3)$

Since $f(x) = x^4 - 2x^2 + x + 3$ is a polynomial function, it is continuous at every number. So the limit as x approaches 1 is the same as the value of $f(x)$ at $x = 1$.

$$\begin{aligned} \lim_{x \to 1} (x^4 - 2x^2 + x + 3) &= 1^4 - 2 \cdot 1^2 + 1 + 3 && \textit{Replace } x \textit{ with } 1. \\ &= 1 - 2 + 1 + 3 \\ &= 3 \end{aligned}$$

The limit of $x^4 - 2x^2 + x + 3$ as x approaches 1 is 3.

b. $\lim_{x \to \pi} \frac{1 - \cos x}{x}$

Since the denominator of $\frac{1 - \cos x}{x}$ is not 0 at $x = \pi$, the function is continuous at $x = \pi$.

$$\begin{aligned} \lim_{x \to \pi} \frac{1 - \cos x}{x} &= \frac{1 - \cos \pi}{\pi} && \textit{Replace } x \textit{ with } \pi. \\ &= \frac{1 - (-1)}{\pi} \text{ or } \frac{2}{\pi} && \cos \pi = -1 \end{aligned}$$

The limit of $\frac{1 - \cos x}{x}$ as x approaches π is $\frac{2}{\pi}$.

Example 2 **Evaluate** $\lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4}$.

$$\begin{aligned} \lim_{x \to 4} \frac{x^2 - 9x + 20}{x - 4} &= \lim_{x \to 4} \frac{(x - 5)(x - 4)}{(x - 4)} && \textit{Factor.} \\ &= \lim_{x \to 4} (x - 5) \\ &= 4 - 5 && \textit{Replace } x \textit{ with } 4. \\ &= -1 \end{aligned}$$