9-6

Study Guide

The Complex Plane and Polar Form of Complex Numbers

In the **complex plane,** the real axis is horizontal and the imaginary axis is vertical. The **absolute value** of a complex number is its distance from zero in the complex plane.

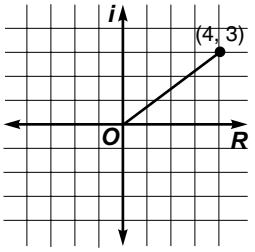
The polar form of the complex number $a + b\boldsymbol{i}$ is $r(\cos\theta + \boldsymbol{i}\sin\theta)$, which is often abbreviated as $r \text{ cis } \theta$. In polar form, r represents the absolute value, or **modulus,** of the complex number. The angle θ is called the **amplitude** or **argument** of the complex number.

Example 1 **Graph each number in the complex plane and find its absolute value.**

a. $\boldsymbol{z = 4 + 3i}$

$|z| = \sqrt{4^2 + 3^2}$

$= 5$

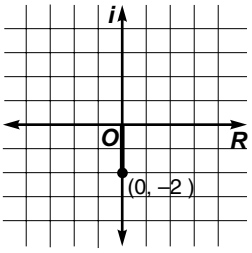


b. $\boldsymbol{z = -2i}$

$z = 0 - 2\boldsymbol{i}$

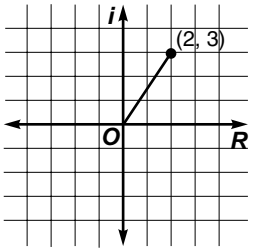
$|z| = \sqrt{0^2 + (-2)^2}$

$= 2$



Example 2 **Express the complex number $2 + 3i$ in polar form.**

First plot the number in the complex plane.



Then find the modulus.

$r = \sqrt{2^2 + 3^2}$ or $\sqrt{13}$

Now find the amplitude. Notice that θ is in Quandrant I.

$\theta = \text{Arctan } \frac{3}{2}$ $\qquad \theta = Arctan\ \frac{b}{a}\ if\ a > 0$

≈ 0.98

Therefore, $2 + 3\boldsymbol{i} \approx \sqrt{13}(\cos 0.98 + \boldsymbol{i}\sin 0.98)$ or $\sqrt{13}$ cis 0.98.