11-3

Study Guide

The Number *e*

The number ***e*** is a special irrational number with an approximate value of 2.718 to three decimal places. The formula for exponential growth or decay is $N = N_0e^{kt}$, where N is the final amount, N_0 is the initial amount, k is a constant, and t is time. The equation $A = Pe^{rt}$, where P is the initial amount, A is the final amount, r is the annual interest rate, and t is time in years, is used for calculating interest that is compounded continuously.

Example 1 ***Demographics*** **The population of Dubuque, Iowa, declined at a rate of 0.4% between 1997 1998. In 1998, the population was 87,806.**

a. Let *t* be the number of years since 1998 and write a function to model the population.

b. Suppose that the rate of decline remains steady at 0.4%. Find the projected population of Dubuque in 2010.

a. $y = ne^{kt}$

$y = 87{,}806e^{-0.004t}$ $\quad n = 87{,}806;\ k = -0.004$

b. In 2010, it will have been 2010 − 1998 or 12 years since the initial population figure. Thus, $t = 12$.

$y = 87{,}806e^{-0.004t}$

$y = 87{,}806e^{-0.004(12)}$ $\quad t = 12$

$y \approx 83690.86531$ $\quad$ *Use a calculator.*

Given a population of 87,806 in 1998 and a steady rate of decline of 0.4%, the population of Dubuque, Iowa, will be approximately 83,691 in 2010.

Example 2 ***Finance*** **Compare the balance after 10 years of a $5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.**

In both cases, $P = 5000$, $r = 0.085$, and $t = 10$. When the interest is compounded quarterly, $n = 4$. Use a calculator to evaluate each expression.

Continuously

$A = Pe^{rt}$

$A = 5000e^{(0.085)(10)}$

$A = 11{,}698.23$

Quarterly

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 5000\left(1 + \frac{0.085}{4}\right)^{4\cdot 10}$

$A = 11{,}594.52$

You would earn $11,698.23 − $11,594.52 = $103.71 more by choosing the account that compounds continuously.