13-6

Study Guide

The Binomial Theorem and Probability

Problems that meet the conditions of a **binomial experiment** can be solved using the binomial expansion. Use the Binomial Theorem to find the probability when the number of trials makes working with the binomial expansion unrealistic.

Example 1 **The probability that Misha will win a word game is $\frac{3}{4}$. If Misha plays the game 5 times, what is the probability that he will win exactly 3 games?**

There are 5 games and each game has only two possible outcomes, win W or lose L. These events are independent and the probability is $\frac{3}{4}$ for each game. So this is a binomial experiment.

When $(W + L)^5$ is expanded, the term W^3L^2 represents 3 wins and 2 losses. The coefficient of W^3L^2 is $C(5, 3)$, or 10.

$$\begin{aligned} P(\text{exactly 3 wins}) &= 10\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 \qquad W = \frac{3}{4}, L = \frac{1}{4} \\ &= 10\left(\frac{27}{64}\right)\left(\frac{1}{16}\right) \\ &= \frac{270}{1024} \\ &= \frac{135}{512} \text{ or about } 26.4\% \end{aligned}$$

Example 2 **The probability that a computer salesperson will make a sale when approaching a customer is $\frac{1}{2}$. If the salesperson approaches 12 customers, what is the probability that 8 sales will be made?**

Let S be the probability of a sale.
Let N be the probability of not making a sale.

$$(S + N)^{12} = \sum \frac{12!}{r!(12 - r)!} P^{12-r} P^{r}$$

Making 8 sales means that 4 sales will not be made. So the probability can be found using the term where $r = 4$.

$$\begin{aligned} \frac{12!}{4!(12 - 4)!}S^8N^4 &= 495S^8N^4 \\ &= 495\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^4 \qquad S = \frac{1}{2}, N = \frac{1}{2} \\ &= \frac{495}{4096} \text{ or } 0.120849609 \end{aligned}$$

The probability of making exactly 8 sales is about 12.1%.