10-3

Study Guide

Ellipses

The standard form of the equation of an **ellipse** is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ when the **major axis** is horizontal. In this case, a^2 is in the denominator of the x term. The standard form is $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ when the major axis is vertical. In this case, a^2 is in the denominator of the y term. In both cases, $c^2 = a^2 - b^2$.

Example **Find the coordinates of the center, the foci, and the vertices of the ellipse with the equation $4x^2 + 9y^2 + 24x - 36y + 36 = 0$. Then graph the equation.**

First write the equation in standard form.

$$4x^2 + 9y^2 + 24x - 36y + 36 = 0$$

$$4(x^2 + 6x + ?) + 9(y^2 - 4y + ?) = -36 + ? + ?$$ *GCF of x terms is 4; GCF of y terms is 9.*

$$4(x^2 + 6x + 9) + 9(y^2 - 4y + 4) = -36 + 4(9) + 9(4)$$ *Complete the square.*

$$4(x+3)^2 + 9(y-2)^2 = 36$$ *Factor.*

$$\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1$$ *Divide each side by 36.*

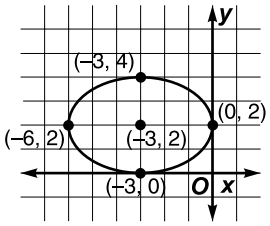
Now determine the values of a, b, c, h, and k. In all ellipses, $a^2 > b^2$. Therefore, $a^2 = 9$ and $b^2 = 4$. Since a^2 is the denominator of the x term, the major axis is parallel to the x-axis.

$a = 3$ $\quad b = 2$ $\quad c = \sqrt{a^2 - b^2}$ or $\sqrt{5}$ $\quad h = -3$ $\quad k = 2$

center: $(-3, 2)$ $\quad (h, k)$
foci: $(-3 \pm \sqrt{5}, 2)$ $\quad (h \pm c, k)$

major axis vertices:
$(0, 2)$ and $(-6, 2)$ $\quad (h \pm a, k)$

minor axis vertices:
$(-3, 4)$ and $(-3, 0)$ $\quad (h, k \pm b)$



Graph these ordered pairs. Then complete the ellipse.