9-4

Study Guide

Polar Form of a Linear Equation

Example 1 **Write the equation $x + 3y = 6$ in polar form.**

The standard form of the equation is $x + 3y - 6 = 0$. To find the values of p and ϕ, write the equation in normal form. To convert to normal form, find the value of $\pm\sqrt{A^2 + B^2}$.

$\pm\sqrt{A^2 + B^2} = \pm\sqrt{1^2 + 3^2}$ or $\pm\sqrt{10}$

Since C is negative, use $+\sqrt{10}$.

The normal form of the equation is
$\frac{1}{\sqrt{10}}x + \frac{3}{\sqrt{10}}y - \frac{6}{\sqrt{10}} = 0$ or $\frac{\sqrt{10}}{10}x + \frac{3\sqrt{10}}{10}y - \frac{3\sqrt{10}}{5} = 0$.

Using the normal form $x \cos \phi + y \sin \phi - p = 0$, we can see that $p = \frac{6}{\sqrt{10}}$ or $\frac{3\sqrt{10}}{5}$. Since $\cos \phi$ and $\sin \phi$ are both positive, the normal lies in Quadrant I.

$\tan \phi = \frac{\sin \phi}{\cos \phi}$

$\tan \phi = 3$ $\qquad \frac{3}{\sqrt{10}} \div \frac{1}{\sqrt{10}} = 3$

$\phi \approx 1.25$ $\qquad$ *Use the Arctangent function.*

Substitute the values for p and ϕ into the polar form.

$p = r \cos(\theta - \phi)$

$\frac{3\sqrt{10}}{5} = r \cos(\theta - 1.25)$ $\qquad$ *Polar form of $x + 3y = 6$*

Example 2 **Write $3 = r \cos(\theta - 30°)$ in rectangular form.**

$3 = r \cos(\theta - 30°)$

$3 = r(\cos \theta \cos 30° + \sin \theta \sin 30°)$ $\qquad$ *Difference identity for cosine*

$3 = r\left(\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta\right)$ $\qquad$ *$\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$*

$3 = \frac{\sqrt{3}}{2}r \cos \theta + \frac{1}{2}r \sin \theta$ $\qquad$ *Distributive property*

$3 = \frac{\sqrt{3}}{2}x + \frac{1}{2}y$ $\qquad$ *$r \cos \theta = x$, $r \sin \theta = y$*

$6 = \sqrt{3}x + y$ $\qquad$ *Multiply each side by 2.*

$0 = \sqrt{3}x + y - 6$ $\qquad$ *Subtract 6 from each side.*

The rectangular form of $3 = r\cos(\theta - 30°)$ is $\sqrt{3}x + y - 6 = 0$.