14-4

Study Guide

The Normal Distribution

A **normal distribution** is a frequency distribution that often occurs when there is a large number of values in a set of data. The graph of a normal distribution is a symmetric, bell-shaped curve known as a **normal curve.** The tables below give the fractional parts of a normally distributed set of data for selected areas about the mean. The letter t represents the number of standard deviations from the mean, that is, $\overline{X} \pm t\sigma$. P represents the fractional part that lies in the interval $\overline{X} \pm t\sigma$.

t	P
0.0	0.000
0.1	0.080
0.2	0.159
0.3	0.236
0.4	0.311
0.5	0.383

t	P
0.6	0.451
0.7	0.516
0.8	0.576
0.9	0.632
1.0	0.683
1.1	0.729

t	P
1.2	0.770
1.3	0.807
1.4	0.838
1.5	0.866
1.6	0.891
1.65	0.900

t	P
1.7	0.911
1.8	0.929
1.9	0.943
1.96	0.950
2.0	0.955
2.1	0.964

t	P
2.2	0.972
2.3	0.979
2.4	0.984
2.5	0.988
2.58	0.990
2.6	0.991

t	P
2.7	0.993
2.8	0.995
2.9	0.996
3.0	0.997
3.5	0.9995
4.0	0.9999

Example 1 **Air passengers traveling through Atlanta have an average layover of 82 minutes with a standard deviation of 7.5 minutes. Sketch a normal curve that represents the frequency of layover times.**

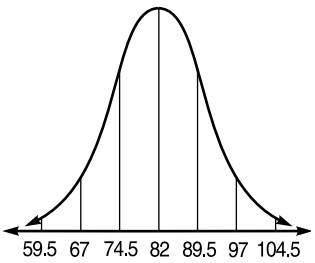
First, find the values defined by the standard deviation in a normal distribution.

$\overline{X} - 1\sigma = 82 - 1(7.5)$ or 74.5 $\quad$ $\overline{X} + 1\sigma = 82 + 1(7.5)$ or 89.5

$\overline{X} - 2\sigma = 82 - 2(7.5)$ or 67 $\quad$ $\overline{X} + 2\sigma = 82 + 2(7.5)$ or 97

$\overline{X} - 3\sigma = 82 - 3(7.5)$ or 59.5 $\quad$ $\overline{X} + 3\sigma = 82 + 3(7.5)$ or 104.5

Then, sketch the general shape of a normal curve. Replace the horizontal scale with the values you have calculated.



Example 2 **Find the upper and lower limits of an interval about the mean within which 15% of the values of a set of normally distributed data can be found if $\overline{X} = 725$ and $\sigma = 4$.**

Use the tables above to find the value of t that most closely approximates $P = 0.15$. For $t = 0.2$, $P = 0.159$. Choose $t = 0.2$.

Now find the limits. $\overline{X} \pm t\sigma = 725 \pm 0.2(4)$ $\quad$ $\overline{X} = 725,\ t = 0.2,\ \sigma = 4$

$= 724.2$ and 725.8

The interval in which 15% of the data lies is 724.2–725.8.