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Study Guide

Modeling Motion Using Parametric Equations

We can use the horizontal and vertical components of a projectile to find parametric equations that represent the path of the projectile.

Example 1 **Find the initial horizontal and vertical velocities of a soccer ball kicked with an initial velocity of 33 feet per second at an angle of 29° with the ground.**

$|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$ $\qquad$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

$|\vec{\mathbf{v}}_x| = 33 \cos 29°$ $\qquad$ $|\vec{\mathbf{v}}_y| = 33 \sin 29°$

$|\vec{\mathbf{v}}_x| \approx 29$ $\qquad$ $|\vec{\mathbf{v}}_y| \approx 16$

The initial horizontal velocity is about 29 feet per second and the initial vertical velocity is about 16 feet per second.

The path of a projectile launched from the ground may be described by the parametric equations $x = t|\vec{\mathbf{v}}| \cos \theta$ for horizontal distance and $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2$ for vertical distance, where t is time and g is acceleration due to gravity. Use $g \approx 9.8$ m/s^2 or 32 ft/s^2.

Example 2 **A rock is tossed at an intitial velocity of 50 meters per second at an angle of 8° with the ground. After 0.8 second, how far has the rock traveled horizontally and vertically?**

First write the position of the rock as a pair of parametric equations defining the postition of the rock for any time t in seconds.

$x = t|\vec{\mathbf{v}}| \cos \theta$ $\qquad$ $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2$

$x = t(50) \cos 8°$ $\qquad$ $y = t(50) \sin 8° - \frac{1}{2}(9.8)t^2$ $\qquad$ $|\vec{v}| = 50\ m/s$

$x = 50t \cos 8°$ $\qquad$ $y = 50t \sin 8° - 4.9t^2$

Then find x and y when $t = 0.8$ second.

$x = 50(0.8) \cos 8°$ $\qquad$ $y = 50(0.8) \sin 8° - 4.9(0.8)^2$

≈ 39.61 $\qquad$ ≈ 2.43

After 0.8 second, the rock has traveled about 39.61 meters horizontally and is about 2.43 meters above the ground.