10-4

Study Guide

Hyperbolas

The standard form of the equation of a **hyperbola** is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ when the **transverse axis** is horizontal, and $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. In both cases, $b^2 = c^2 - a^2$.

Example **Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $25x^2 - 9y^2 + 100x - 54y - 206 = 0$. Then graph the equation.**

Write the equation in standard form.

$$25x^2 - 9y^2 + 100x - 54y - 206 = 0$$

$$25(x^2 + 4x + ?) - 9(y^2 + 6y + ?) = 206 + ? + ?$$ *GCF of x terms is 25; GCF of y terms is 9.*

$$25(x^2 + 4x + 4) - 9(y^2 + 6y + 9) = 206 + 25(4) + (-9)(9)$$ *Complete the square.*

$$25(x + 2)^2 - 9(y + 3)^2 = 225$$ *Factor.*

$$\frac{(x+2)^2}{9} - \frac{(y+3)^2}{25} = 1$$ *Divide each side by 225.*

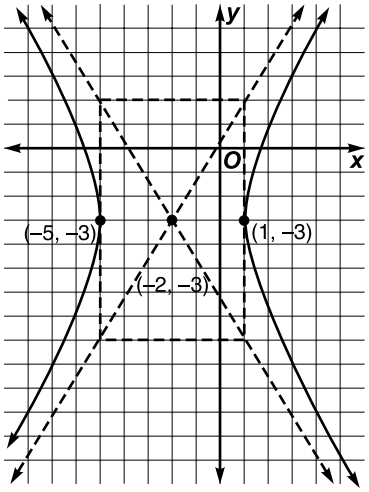
From the equation, $h = -2$, $k = -3$, $a = 3$, $b = 5$, and $c = \sqrt{34}$. The center is at $(-2, -3)$.

Since the x terms are in the first expression, the hyperbola has a horizontal transverse axis.

The vertices are at $(h \pm a, k)$ or $(1, -3)$ and $(-5, -3)$.

The foci are at $(h \pm c, k)$ or $(-2 \pm \sqrt{34}, -3)$.

The equations of the asymptotes are $y - k = \pm\frac{b}{a}(x - h)$ or $y + 3 = \pm\frac{5}{3}(x + 2)$.



Graph the center, the vertices, and the rectangle guide, which is $2a$ units by $2b$ units. Next graph the asymptotes. Then sketch the hyperbola.