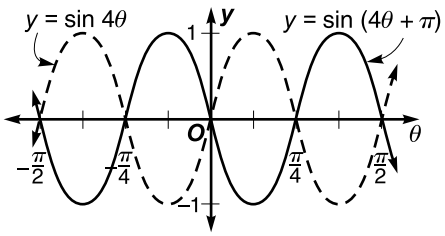
6-5

Study Guide

Translations of Sine and Cosine Functions

A horizontal translation of a trigonometric function is called a **phase shift**. The phase shift of the functions $y = A \sin (k\theta + c)$ and $y = A \cos (k\theta + c)$ is $-\frac{c}{k}$, where $k > 0$. If $c > 0$, the shift is to the left. If $c < 0$, the shift is to the right. The **vertical shift** of the functions $y = A \sin (k\theta + c) + h$ and $y = A \cos (k\theta + c) + h$ is h. If $h > 0$, the shift is upward. If $h < 0$, the shift is downward. The **midline** about which the graph oscillates is $y = h$.

Example 1 **State the phase shift for $y = \sin (4\theta + \pi)$. Then graph the function.**

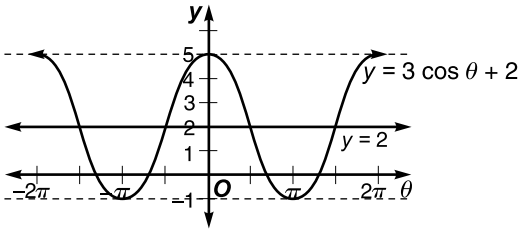


The phase shift of the function is $-\frac{c}{k}$ or $-\frac{\pi}{4}$.

To graph $y = \sin (4\theta + \pi)$, consider the graph of $y = \sin 4\theta$. The graph of $y = \sin 4\theta$ has an amplitude of 1 and a period of $\frac{\pi}{2}$. Graph this function, then shift the graph $-\frac{\pi}{4}$.

Example 2 **State the vertical shift and the equation of the midline for $y = 3 \cos \theta + 2$. Then graph the function.**

The vertical shift is 2 units upward. The midline is the graph of $y = 2$.



To graph the function, draw the midline. Since the amplitude of the function is $|3|$, or 3, draw dashed lines parallel to the midline which are 3 units above and below $y = 2$. That is, $y = 5$ and $y = -1$. Then draw the cosine curve with a period of 2π.

Example 3 **Write an equation of the cosine function with amplitude 2.9, period $\frac{2\pi}{5}$, phase shift $-\frac{\pi}{2}$, and vertical shift -3.**

The form of the equation will be $y = A \cos (k\theta + c) + h$. Find the values of A, k, c, and h.

A: $|A| = 2.9$
$A = 2.9$ or -2.9

k: $\frac{2\pi}{k} = \frac{2\pi}{5}$ *The period is* $\frac{2\pi}{5}$.
$k = 5$

c: $-\frac{c}{k} = -\frac{\pi}{2}$ *The phase shift is* $-\frac{\pi}{2}$.
$-\frac{c}{5} = -\frac{\pi}{2}$ $k = 5$
$c = \frac{5\pi}{2}$

h: $h = -3$

The possible equations are $y = \pm 2.9 \cos \left(5\theta + \frac{5\pi}{2}\right) - 3$.