12-4

Study Guide

Convergent and Divergent Series

If an infinite series has a sum, or limit, the series is **convergent.** If a series is not convergent, it is **divergent.** When a series is neither arithmetic nor geometric and all the terms are positive, you can use the **ratio test** or the **comparison test** to determine whether the series is convergent or divergent.

Ratio Test	Let a_n and a_{n+1} represent two consecutive terms of a series of positive terms. Suppose $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and $r = \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$. The series is convergent if $r < 1$ and divergent if $r > 1$. If $r = 1$, the test provides no information.

Comparison Test	• A series of positive terms is convergent if, for $n > 1$, each term of the series is equal to or less than the value of the corresponding term of some convergent series of positive terms. • A series of positive terms is divergent if, for $n > 1$, each term of the series is equal to or greater than the value of the corresponding term of some divergent series of positive terms.

Example 1 **Use the ratio test to determine whether the series $\frac{1\cdot 2}{2^1} + \frac{2\cdot 3}{2^2} + \frac{3\cdot 4}{2^3} + \frac{4\cdot 5}{2^4} + \cdots$ is convergent or divergent.**

First, find the nth term. Then use the ratio test.

$a_n = \frac{n(n+1)}{2^n}$ $\qquad a_{n+1} = \frac{(n+1)(n+2)}{2^{n+1}}$

$$r = \lim_{n\to\infty} \frac{\frac{(n+1)(n+2)}{2^{n+1}}}{\frac{n(n+1)}{2^n}}$$

$$r = \lim_{n\to\infty} \frac{(n+1)(n+2)}{2^{n+1}} \cdot \frac{2^n}{n(n+1)}$$ *Multiply by the reciprocal of the divisor.*

$$r = \lim_{n\to\infty} \frac{n+2}{2n}$$ $\frac{2^n}{2^{n+1}} = \frac{1}{2}$

$$r = \lim_{n\to\infty} \frac{1+\frac{2}{n}}{2}$$ *Divide by the highest power of n and apply limit theorems.*

$$r = \frac{1}{2}$$ *Since $r < 1$, the series is convergent.*

Example 2 **Use the comparison test to determine whether the series $\frac{1}{4^2} + \frac{1}{7^2} + \frac{1}{10^2} + \frac{1}{13^2} + \cdots$ is convergent or divergent.**

The general term of the series is $\frac{1}{(3n+1)^2}$. The general term of the convergent series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$ is $\frac{1}{n^2}$. Since $\frac{1}{(3n+1)^2} < \frac{1}{n^2}$ for all $n \geq 1$, the series $\frac{1}{4^2} + \frac{1}{7^2} + \frac{1}{10^2} + \frac{1}{13^2} + \cdots$ is also convergent.