2-4

Study Guide

Modeling Motion with Matrices

You can use matrices to perform many **transformations,** such as **translations** (slides), **reflections** (flips), **rotations** (turns), and **dilations** (enlargements or reductions).

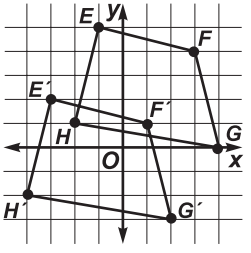
Example 1 **Suppose quadrilateral *EFGH* with vertices *E*(−1, 5), *F*(3, 4), *G*(4, 0), and *H*(−2, 1) is translated 2 units left and 3 units down.**

The **vertex matrix** for the quadrilateral is $\begin{bmatrix} -1 & 3 & 4 & -2 \\ 5 & 4 & 0 & 1 \end{bmatrix}$.

The **translation matrix** is $\begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix}$.

Adding the two matrices gives the coordinates of the vertices of the translated quadrilateral.

$$\begin{bmatrix} -1 & 3 & 4 & -2 \\ 5 & 4 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & -2 & -2 & -2 \\ -3 & -3 & -3 & -3 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 2 & -4 \\ 2 & 1 & -3 & -2 \end{bmatrix}$$



Graphing the **pre-image** and the **image** of the translated quadrilateral on the same axes, we can see the effect of the translation.

The chart below summarizes the matrices needed to produce specific reflections or rotations. All rotations are counterclockwise about the origin.

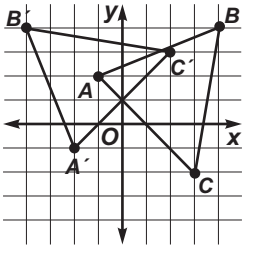
Reflections	$R_{x\text{-axis}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$R_{y\text{-axis}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$R_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Rotations	$Rot_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$Rot_{180} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$Rot_{270} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Example 2 **A triangle has vertices *A*(−1, 2), *B*(4, 4), and *C*(3, −2). Find the coordinates of the image of the triangle after a rotation of 90° counterclockwise about the origin.**

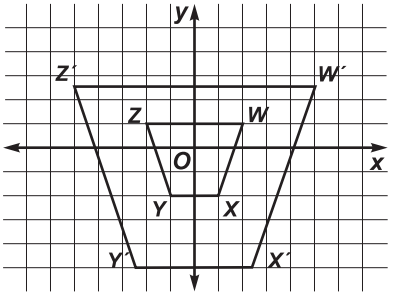
The vertex matrix is $\begin{bmatrix} -1 & 4 & 3 \\ 2 & 4 & -2 \end{bmatrix}$.

Multiply it by the 90° **rotation matrix.**

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 4 & 3 \\ 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -4 & 2 \\ -1 & 4 & 3 \end{bmatrix}$$



Example 3 **Trapezoid *WXYZ* has vertices *W*(2, 1), *X*(1, −2), *Y*(−1, −2), and *Z*(−2, 1). Find the coordinates of dilated trapezoid *W′X′Y′Z′* for a scale factor of 2.5.**

Perform scalar multiplication on the vertex matrix for the trapezoid.

$$2.5\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2.5 & -2.5 & -5 \\ 2.5 & -5 & -5 & 2.5 \end{bmatrix}$$