11-4

Study Guide

Logarithmic Functions

In the function $x = a^y$, y is called the **logarithm** of x. It is usually written as $y = \log_a x$ and is read "y equals the log, base a, of x." Knowing that if $a^u = a^v$ then $u = v$, you can evaluate a logarithmic expression to determine its logarithm.

Example 1 **Write $\log_7 49 = 2$ in exponential form.**

The base is 7 and the exponent is 2.
$7^2 = 49$

Example 2 **Write $2^5 = 32$ in logarithmic form.**

The base is 2, and the exponent or logarithm is 5.
$\log_2 32 = 5$

Example 3 **Evaluate the expression $\log_5 \frac{1}{25}$.**

Let $x = \log_5 \frac{1}{25}$.

$x = \log_5 \frac{1}{25}$	
$5^x = \frac{1}{25}$	*Definition of logarithm.*
$5^x = (25)^{-1}$	$a^{-m} = \frac{1}{a^m}$
$5^x = (5^2)^{-1}$	$5^2 = 25$
$5^x = 5^{-2}$	$(a^m)^n = a^{mn}$
$x = -2$	*If $a^u = a^v$, then $u = v$.*

Example 4 **Solve each equation.**

a. $\log_6 (4x + 6) = \log_6 (8x - 2)$

$\log_6 (4x + 6) = \log_6 (8x - 2)$	
$4x + 6 = 8x - 2$	*If $\log_b m = \log_b n$, then $m = n$.*
$-4x = -8$	
$x = 2$	

b. $\log_9 x + \log_9 (x - 2) = \log_9 3$

$\log_9 x + \log_9 (x - 2) = \log_9 3$	
$\log_9 [x(x - 2)] = \log_9 3$	$\log_b mn = \log_b m + \log_b n$
$x^2 - 2x = 3$	*If $\log_b m = \log_b n$, then $m = n$.*
$x^2 - 2x - 3 = 0$	
$(x - 3)(x + 1) = 0$	*Factor.*
$x - 3 = 0$ or $x + 1 = 0$	*Find the zeros.*
$x = 3$ or $x = -1$.	

The log of a negative value does not exist, so the answer is $x = 3$.