7-7

Study Guide

Distance from a Point to a Line

The distance from a point at (x_1, y_1) to a line with equation $Ax + By + C = 0$ can be determined by using the formula $d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$. The sign of the radical is chosen opposite the sign of C.

Example 1 **Find the distance between $P(3, 4)$ and the line with equation $4x + 2y = 10$.**

First, rewrite the equation of the line in standard form.

$$4x + 2y - 10 = 0$$

Then, use the formula for the distance from a point to a line.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{4(3) + 2(4) - 10}{\pm\sqrt{4^2 + 2^2}}$ $\quad$ *$A = 4, B = 2, C = -10, x_1 = 3, y_1 = 4$*

$d = \frac{10}{2\sqrt{5}}$ or $\sqrt{5}$ $\quad$ *Since C is negative, use $+\sqrt{A^2 + B^2}$.*

$d \approx 2.24$ units

Therefore, P is approximately 2.24 units from the line with equation $4x + 2y = 10$. Since d is positive, P is on the opposite side of the line from the origin.

You can also use the formula to find the distance between two parallel lines. To do this, choose any point on one of the lines and use the formula to find the distance from that point to the other line.

Example 2 **Find the distance between the lines with equations $2x - 2y = 5$ and $y = x - 1$.**

Since $y = x - 1$ is in slope-intercept form, you can see that it passes through the point at $(0, -1)$. Use this point to find the distance to the other line.

The standard form of the other equation is $2x - 2y - 5 = 0$.

$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$

$d = \frac{2(0) - 2(-1) - 5}{\pm\sqrt{2^2 + (-2)^2}}$ $\quad$ *$A = 2, B = -2, C = -5, x_1 = 0, y_1 = -1$*

$d = -\frac{3}{2\sqrt{2}}$ or $-\frac{3\sqrt{2}}{4}$ $\quad$ *Since C is negative, use $+\sqrt{A^2 + B^2}$.*

≈ -1.06

The distance between the lines is about 1.06 units.