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Study Guide

Normal Form of a Linear Equation

Normal Form	The normal form of a linear equation is $x \cos \phi + y \sin \phi - p = 0$, where p is the length of the normal from the line to the origin and ϕ is the positive angle formed by the positive x-axis and the normal.

You can write the standard form of a linear equation if you are given the values of ϕ and p.

Example 1 **Write the standard form of the equation of a line for which the length of the normal segment to the origin is 5 and the normal makes an angle of 135° with the positive *x*-axis.**

$$x \cos \phi + y \sin \phi - p = 0 \quad \textit{Normal form}$$
$$x \cos 135° + y \sin 135° - 5 = 0 \quad \phi = 135° \textit{ and } p = 5$$
$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - 5 = 0$$
$$\sqrt{2}x - \sqrt{2}y + 10 = 0 \quad \textit{Multiply each side by } -2.$$

The standard form of the equation is $\sqrt{2}x - \sqrt{2}y + 10 = 0$.

The standard form of a linear equation, $Ax + By + C = 0$, can be changed to the normal form by dividing each term of the equation by $\pm\sqrt{A^2 + B^2}$. The sign is chosen opposite the sign of C. You can then find the length of the normal, p units, and the angle ϕ.

Example 2 **Write $3x + 4y - 10 = 0$ in normal form. Then find the length of the normal and the angle it makes with the positive *x*-axis.**

Since C is negative, use $\sqrt{A^2 + B^2}$ to determine the normal form.

$\sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2}$ or 5

The normal form is $\frac{3}{5}x + \frac{4}{5}y - \frac{10}{5} = 0$ or $\frac{3}{5}x + \frac{4}{5}y - 2 = 0$.

Therefore, $\cos \phi = \frac{3}{5}$, $\sin \phi = \frac{4}{5}$, and $p = 2$.

Since $\cos \phi$ and $\sin \phi$ are both positive, ϕ must lie in Quadrant I.

$\tan \phi = \frac{\frac{4}{5}}{\frac{3}{5}}$ or $\frac{4}{3}$ $\qquad \tan \phi = \frac{\sin \phi}{\cos \phi}$

$\phi \approx 53°$

The normal segment has length 2 units and makes an angle of 53° with the positive x-axis.