6-7

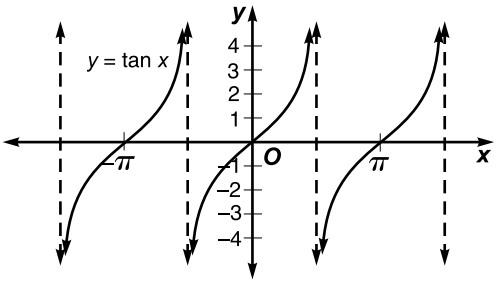
Study Guide

Graphing Other Trigonometric Functions

The period of functions $y = \csc k\theta$ and $y = \sec k\theta$ is $\frac{2\pi}{k}$, where $k > 0$. The period of functions $y = \tan k\theta$ and $y = \cot k\theta$ is $\frac{\pi}{k}$, where $k > 0$. The phase shift and vertical shift work the same way for all trigonometric functions. For example, the phase shift of the function $y = \tan (k\theta + c) + h$ is $-\frac{c}{k}$, and its vertical shift is h.

Example 1 **Graph $y = \tan x$.**

To graph $y = \tan x$, first draw the asymptotes located at $x = \frac{\pi}{2}n$, where n is an odd integer. Then plot the following coordinate pairs and draw the curves.



$\left(-\frac{5\pi}{4}, -1\right)$, $(-\pi, 0)$, $\left(-\frac{3\pi}{4}, 1\right)$, $\left(-\frac{\pi}{4}, -1\right)$, $(0, 0)$, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, -1\right)$, $(\pi, 0)$, $\left(\frac{5\pi}{4}, 1\right)$

Notice that the range values for the interval $-\frac{3\pi}{2} \le x \le -\frac{\pi}{2}$ repeat for the intervals $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$. So, the tangent function is a periodic function with a period of $\frac{\pi}{k}$ or π.

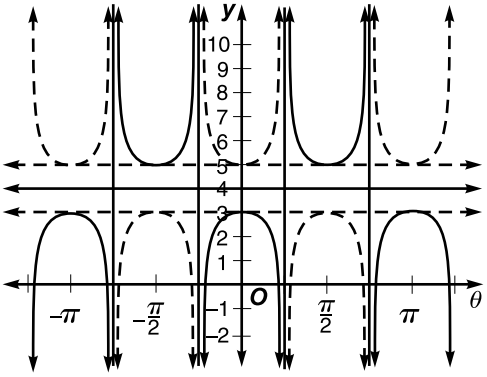
Example 2 **Graph $y = \sec (2\theta + \pi) + 4$.**

Since $k = 2$, the period is $\frac{2\pi}{2}$ or π. Since $c = \pi$, the phase shift is $-\frac{\pi}{2}$. The vertical shift is 4.

Using this information, follow the steps for graphing a secant function.

Step 1 Draw the midline, which is the graph of $y = 4$.

Step 2 Draw dashed lines parallel to the midline, which are 1 unit above and below $y = 4$.

Step 3 Draw the secant curve with a period of π.

Step 4 Shift the graph $\frac{\pi}{2}$ units to the left.