2-6

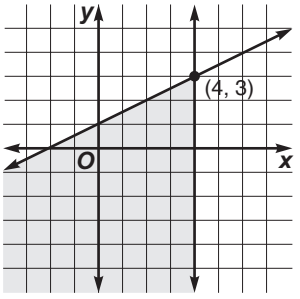
Study Guide

Solving Systems of Linear Inequalities

To solve a **system of linear inequalities**, you must find the ordered pairs that satisfy all inequalities. One way to do this is to graph the inequalities on the same coordinate plane. The intersection of the graphs contains points with ordered pairs in the solution set. If the graphs of the inequalities do not intersect, then the system has no solution.

Example 1 **Solve the system of inequalities by graphing.** $-x + 2y \leq 2$, $x \leq 4$

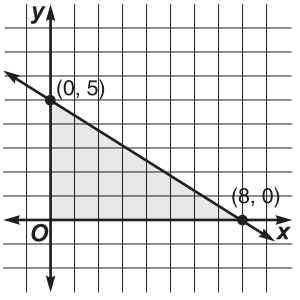
The shaded region represents the solution to the system of inequalities.



A system of more than two linear inequalities can have a solution that is a bounded set of points called a **polygonal convex set.**

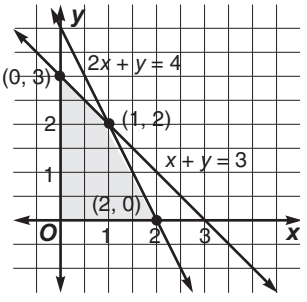
Example 2 **Solve the system of inequalities by graphing and name the coordinates of the vertices of the polygonal convex set.** $x \geq 0$, $y \geq 0$, $5x + 8y \leq 40$

The region shows points that satisfy all three inequalities. The region is a triangle whose vertices are the points at (0, 0), (0, 5), and (8, 0).



Example 3 **Find the maximum and minimum values of $f(x, y) = x + 2y + 1$ for the polygonal convex set determined by the following inequalities.**
$x \geq 0 \quad y \geq 0 \quad 2x + y \leq 4 \quad x + y \leq 3$

First, graph the inequalities and find the coordinates of the vertices of the resulting polygon.



The coordinates of the vertices are (0, 0), (2, 0), (1, 2), and (0, 3).

Then, evaluate the function $f(x, y) = x + 2y + 1$ at each vertex.

$f(0, 0) = 0 + 2(0) + 1 = 1$ $\qquad$ $f(1, 2) = 1 + 2(2) + 1 = 6$

$f(2, 0) = 2 + 2(0) + 1 = 3$ $\qquad$ $f(0, 3) = 0 + 2(3) + 1 = 7$

Thus, the maximum value of the function is 7, and the minimum value is 1.