9-5

Study Guide

Simplifying Complex Numbers

Add and subtract complex numbers by performing the chosen operation on both the **real** and **imaginary parts.** Find the product of two or more complex numbers by using the same procedures used to multiply binomials. To simplify the quotient of two complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator.

To find the value of i^n, let R be the remainder when n is divided by 4.	
if R = 0	$i^n = 1$
if R = 1	$i^n = i$
if R = 2	$i^n = -1$
if R = 3	$i^n = -i$

Powers of i	
$i^1 = i$	$i^2 = -1$
$i^3 = i^2 \cdot i = -i$	$i^4 = (i^2)^2 = 1$
$i^5 = i^4 \cdot i = i$	$i^6 = i^4 \cdot i^2 = -1$
$i^7 = i^4 \cdot i^3 = -i$	$i^8 = (i^2)^4 = 1$

Example 1 **Simplify each power of i.**

a. i^{30}

Method 1
$30 \div 4 = 7$ R 2
If R = 2, $i^n = -1$.
$i^{30} = -1$

Method 2
$i^{30} = (i^4)^7 \cdot i^2$
$= (1)^7 \cdot i^2$
$= -1$

b. i^{-11}

Method 1
$-11 \div 4 = -3$ R 1
If R = 1, $i^n = i$.
$i^{-11} = i$

Method 2
$i^{-11} = (i^4)^{-3} \cdot i^1$
$= (1)^{-3} \cdot i^1$
$= i$

Example 2 **Simplify each expression.**

a. $(3 + 2i) + (5 - 3i)$

$(3 + 2i) + (5 - 3i)$
$= (3 + 5) + (2i - 3i)$
$= 8 - i$

b. $(8 - 4i) - (9 - 7i)$

$(8 - 4i) - (9 - 7i)$
$= 8 - 4i - 9 + 7i$
$= -1 + 3i$

Example 3 **Simplify $(4 - 2i)(5 - 3i)$.**

$(4 - 2i)(5 - 3i) = 5(4 - 2i) - 3i(4 - 2i)$ *Distributive property*
$= 20 - 10i - 12i + 6i^2$ *Distributive property*
$= 20 - 10i - 12i + 6(-1)$ $i^2 = -1$
$= 14 - 22i$

Example 4 **Simplify $(4 - 5i) \div (2 + i)$.**

$(4 - 5i) \div (2 + i) = \dfrac{4 - 5i}{2 + i}$
$= \dfrac{4 - 5i}{2 + i} \cdot \dfrac{2 - i}{2 - i}$ *$2 - i$ is the conjugate of $2 + i$.*
$= \dfrac{8 - 10i - 4i + 5i^2}{4 - i^2}$
$= \dfrac{8 - 14i + 5(-1)}{4 - (-1)}$ $i^2 = -1$
$= \dfrac{3 - 14i}{5}$
$= \dfrac{3}{5} - \dfrac{14}{5}i$ *Write the answer in the form $a + bi$.*