2-5

Study Guide

Determinants and Multiplicative Inverses of Matrices

Each square matrix has a **determinant.** The determinant of a 2×2 matrix is a number denoted by $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ or $\det\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Its value is $a_1b_2 - a_2b_1$.

Example 1 **Find the value of $\begin{vmatrix} 5 & -2 \\ 3 & 1 \end{vmatrix}$.**

$$\begin{vmatrix} 5 & -2 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(-2) \text{ or } 11$$

The **minor** of an element can be found by deleting the row and column containing the element.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The minor of a_1 is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$.

Example 2 **Find the value of $\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix}$.**

$$\begin{vmatrix} 8 & 9 & 3 \\ 3 & 5 & 7 \\ -1 & 2 & 4 \end{vmatrix} = 8\begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} - 9\begin{vmatrix} 3 & 7 \\ -1 & 4 \end{vmatrix} + 3\begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix}$$
$$= 8(6) - 9(19) + 3(11) \text{ or } -90$$

The multiplicative inverse of a matrix is defined as follows.

$$\text{If } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \text{ and } \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0, \text{ then } A^{-1} = \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}.$$

Example 3 **Solve the system of equations by using matrix equations.**

$$\mathbf{5x + 4y = -3}$$
$$\mathbf{3x - 5y = -24}$$

Write the system as a matrix equation.

$$\begin{bmatrix} 5 & 4 \\ 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$

To solve the matrix equation, first find the inverse of the coefficient matrix.

$$\overset{1}{\overline{\begin{vmatrix} 5 & 4 \\ 3 & -5 \end{vmatrix}}}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} = -\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix}$$

Now multiply each side of the matrix equation by the inverse and solve.

$$-\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 4 \\ 3 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$
$$= -\frac{1}{37}\begin{bmatrix} -5 & -4 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -24 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

The solution is $(-3, 3)$.