2-2

Study Guide

Solving Systems of Equations in Three Variables

You can solve systems of three equations in three variables using the same algebraic techniques you used to solve systems of two equations.

Example **Solve the system of equations by elimination.**

$$\mathbf{4x + y + 2z = -8}$$
$$\mathbf{2x - 3y - 4z = 4}$$
$$\mathbf{-7x + 2y + 3z = 2}$$

Choose a pair of equations and then eliminate one of the variables. Because the coefficient of y is 1 in the first equation, eliminate y from the second and third equations.

To eliminate y using the first and second equations, multiply both sides of the first equation by 3.

$$3(4x + y + 2z) = 3(-8)$$
$$12x + 3y + 6z = -24$$

Then add the second equation to that result.

$$\begin{array}{r} 12x + 3y + 6z = -24 \\ 2x - 3y - 4z = 4 \\ \hline 14x + 2z = -20 \end{array}$$

To eliminate y using the first and third equations, multiply both sides of the first equation by -2.

$$-2(4x + y + 2z) = -2(-8)$$
$$-8x - 2y - 4z = 16$$

Then add the third equation to that result.

$$\begin{array}{r} -8x - 2y - 4z = 16 \\ -7x + 2y + 3z = 2 \\ \hline -15x - z = 18 \end{array}$$

Now you have two linear equations in two variables. Solve this system. Eliminate z by multiplying both sides of the second equation by 2. Then add the two equations.

$$2(-15x - z) = 2(18)$$
$$-30x - 2z = 36$$

$$\begin{array}{r} 14x + 2z = -20 \\ -30x - 2z = 36 \\ \hline -16x = 16 \\ x = -1 \end{array}$$

The value of x is -1.

By substituting the value of x into one of the equations in two variables, we can solve for the value of z.

$$14x + 2z = -20$$
$$14(-1) + 2z = -20 \qquad x = -1$$
$$z = -3$$

The value of z is -3.

Finally, use one of the original equations to find the value of y.

$$4x + y + 2z = -8$$
$$4(-1) + y + 2(-3) = -8 \qquad x = -1,\ z = -3$$
$$y = 2$$

The value of y is 2.

The solution is $x = -1$, $y = 2$, $z = -3$. This can be written as the **ordered triple** $(-1, 2, -3)$.