12-2

Study Guide

Geometric Sequences and Series

A **geometric sequence** is a sequence in which each term after the first, a_1, is the product of the preceding term and the **common ratio**, r. The terms between two nonconsecutive terms of a geometric sequence are called **geometric means.** The indicated sum of the terms of a geometric sequence is a **geometric series.**

Example 1 **Find the 7th term of the geometric sequence 157, −47.1, 14.13, ...**

First, find the common ratio.
$a_2 \div a_1 = -47.1 \div 157$ or -0.3
The common ratio is -0.3.

Then, use the formula for the nth term of a geometric sequence.
$a_n = a_1 r^{n-1}$
$a_7 = 157(-0.3)^6$ $\quad n = 7, a_1 = 157, r = -0.3$
$a_7 = 0.114453$

The 7th term is 0.114453.

Example 2 **Write a sequence that has two geometric means between 6 and 162.**
The sequence will have the form 6, _?_, _?_, 162.

First, find the common ratio.
$a_n = a_1 r^{n-1}$
$162 = 6r^3$ $\quad a_4 = 162, a_1 = 6, n = 4$
$27 = r^3$ $\quad$ *Divide each side by 6.*
$3 = r$ $\quad$ *Take the cube root of each side.*

Then, determine the geometric sequence.
$a_2 = 6 \cdot 3$ or 18 $\qquad a_3 = 18 \cdot 3$ or 54
The sequence is 6, 18, 54, 162.

Example 3 **Find the sum of the first twelve terms of the geometric series $12 - 12\sqrt{2} + 24 - 24\sqrt{2} + \cdots$.**

First, find the common ratio.
$a_2 \div a_1 = -12\sqrt{2} \div 12$ or $-\sqrt{2}$
The common ratio is $-\sqrt{2}$.

$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$

$S_{12} = \dfrac{12 - 12(-\sqrt{2})^{12}}{1 - (-\sqrt{2})}$ $\qquad n = 12, a_1 = 12, r = -\sqrt{2}$

$S_{12} = 756(1 - \sqrt{2})$ $\qquad$ *Simplify.*

The sum of the first twelve terms of the series is $756(1 - \sqrt{2})$.