3-8

Study Guide

Direct, Inverse, and Joint Variation

A **direct variation** can be described by the equation $y = kx^n$. The k in this equation is called the **constant of variation.** To express a direct variation, we say that y varies directly as x^n. An inverse variation can be described by the equation $y = \frac{k}{x^n}$ or $x^n y = k$. When quantities are **inversely proportional,** we say they *vary inversely* with each other. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities and can be described by the equation $y = kx^n z^n$.

Example 1 **Suppose y varies directly as x and $y = 14$ when $x = 8$.**

a. Find the constant of variation and write an equation of the form $y = kx^n$.

b. Use the equation to find the value of y when $x = 4$.

a. The power of x is 1, so the direct variation equation is $y = kx$.

$y = kx$

$14 = k(8)$ *$y = 14$, $x = 8$*

$1.75 = k$ *Divide each side by 8.*

The constant of variation is 1.75. The equation relating x and y is $y = 1.75x$.

b. $y = 1.75x$

$y = 1.75(4)$ *$x = 4$*

$y = 7$

When $x = 4$, the value of y is 7.

Example 2 **If y varies inversely as x and $y = 102$ when $x = 7$, find x when $y = 12$.**

Use a proportion that relates the values.

$\frac{x_1{}^n}{y_2} = \frac{x_2{}^n}{y_1}$

$\frac{7}{12} = \frac{x}{102}$ *Substitute the known values.*

$12x = 714$ *Cross multiply.*

$x = \frac{714}{12}$ or 59.5 *Divide each side by 12.*

When $y = 12$, the value of x is 59.5.