4-3

Study Guide

The Remainder and Factor Theorems

The Remainder Theorem	If a polynomial $P(x)$ is divided by $x - r$, the remainder is a constant $P(r)$, and $P(x) = (x - r) \cdot Q(x) + P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of $P(x)$.

Example 1 **Divide $x^4 - 5x^2 - 17x - 12$ by $x + 3$.**

$$
\begin{array}{r}
x^3 - 3x^2 + 4x - 29 \\
x + 3 \overline{)\, x^4 + 0x^3 - 5x^2 - 17x - 12} \\
\underline{x^4 + 3x^3 \qquad\qquad\qquad} \\
-3x^3 - 5x^2 \qquad\qquad \\
\underline{-3x^3 - 9x^2 \qquad\qquad} \\
4x^2 - 17x \qquad \\
\underline{4x^2 + 12x \qquad} \\
-29x - 12 \\
\underline{-29x - 87} \\
75 \leftarrow \textit{remainder}
\end{array}
$$

Find the value of r in this division.

$x - r = x + 3$

$-r = 3$

$r = -3$

According to the Remainder Theorem, $P(r)$ or $P(-3)$ should equal 75.

Use the Remainder Theorem to check the remainder found by long division.

$$
\begin{aligned}
P(x) &= x^4 - 5x^2 - 17x - 12 \\
P(-3) &= (-3)^4 - 5(-3)^2 - 17(-3) - 12 \\
&= 81 - 45 + 51 - 12 \text{ or } 75
\end{aligned}
$$

The Factor Theorem is a special case of the Remainder Theorem and can be used to quickly test for factors of a polynomial.

The Factor Theorem	The binomial $x - r$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.

Example 2 **Use the Remainder Theorem to find the remainder when $2x^3 + 5x^2 - 14x - 8$ is divided by $x - 2$. State whether the binomial is a factor of the polynomial. Explain.**

Find $f(2)$ to see if $x - 2$ is a factor.

$$
\begin{aligned}
f(x) &= 2x^3 + 5x^2 - 14x - 8 \\
f(2) &= 2(2)^3 + 5(2)^2 - 14(2) - 8 \\
&= 16 + 20 - 28 - 8 \\
&= 0
\end{aligned}
$$

Since $f(2) = 0$, the remainder is 0. So the binomial $x - 2$ is a factor of the polynomial by the Factor Theorem.