1-2

Study Guide

Composition of Functions

Operations of Functions	Two functions can be added together, subtracted, multiplied, or divided to form a new function.

Example 1 **Given $f(x) = x^2 - x - 6$ and $g(x) = x + 2$, find each function.**

a. $(f + g)(x)$

$$\begin{aligned}(f + g)(x) &= f(x) + g(x)\\ &= x^2 - x - 6 + x + 2\\ &= x^2 - 4\end{aligned}$$

b. $(f - g)(x)$

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= x^2 - x - 6 - (x + 2)\\ &= x^2 - 2x - 8\end{aligned}$$

c. $(f \cdot g)(x)$

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x)\\ &= (x^2 - x - 6)(x + 2)\\ &= x^3 + x^2 - 8x - 12\end{aligned}$$

d. $\left(\frac{f}{g}\right)(x)$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{x^2 - x - 6}{x + 2}\\ &= \frac{(x - 3)(x + 2)}{x + 2}\\ &= x - 3,\ x \neq -2\end{aligned}$$

Functions can also be combined by using **composition**. The function formed by composing two functions f and g is called the **composite** of f and g, and is denoted by $f \circ g$. $[f \circ g](x)$ is found by substituting $g(x)$ for x in $f(x)$.

Example 2 **Given $f(x) = 3x^2 + 2x - 1$ and $g(x) = 4x + 2$, find $[f \circ g](x)$ and $[g \circ f](x)$.**

$$\begin{aligned}[f \circ g](x) &= f(g(x))\\ &= f(4x + 2) && \textit{Substitute } 4x + 2 \textit{ for } g(x).\\ &= 3(4x + 2)^2 + 2(4x + 2) - 1 && \textit{Substitute } 4x + 2 \textit{ for } x \textit{ in } f(x).\\ &= 3(16x^2 + 16x + 4) + 8x + 4 - 1\\ &= 48x^2 + 56x + 15\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g(f(x))\\ &= g(3x^2 + 2x - 1) && \textit{Substitute } 3x^2 + 2x - 1 \textit{ for } f(x).\\ &= 4(3x^2 + 2x - 1) + 2 && \textit{Substitute } 3x^2 + 2x - 1 \textit{ for } x \textit{ in } g(x).\\ &= 12x^2 + 8x - 2\end{aligned}$$