8-6

Study Guide

Vectors and Parametric Equations

Vector equations and **parametric equations** allow us to model movement.

Example 1 **Write a vector equation describing a line passing through $P_1(8, 4)$ and parallel to $\vec{\mathbf{a}} = \langle 6, -1\rangle$. Then write parametric equations of the line.**

Let the line ℓ through $P_1(8, 4)$ be parallel to $\vec{\mathbf{a}}$. For any point $P_2(x, y)$ on ℓ, $P_1P_2\langle x - 8, y - 4\rangle$. Since P_1P_2 is on ℓ and is parallel to $\vec{\mathbf{a}}$, $P_1P_2 = t\vec{\mathbf{a}}$, for some value t. By substitution, we have $\langle x - 8, y - 4\rangle = t\langle 6, -1\rangle$.

Therefore, the equation $\langle x - 8, y - 4\rangle = t\langle 6, -1\rangle$ is a vector equation describing all of the points (x, y) on ℓ parallel to $\vec{\mathbf{a}}$ through $P_1(8, 4)$.

Use the general form of the parametric equations of a line with $\langle a_1, a_2\rangle = \langle 6, -1\rangle$ and $\langle x_1, y_1\rangle = \langle 8, 4\rangle$.

$$\begin{aligned} x &= x_1 + ta_1 & \qquad y &= y_1 + ta_2 \\ x &= 8 + t(6) & y &= 4 + t(-1) \\ x &= 8 + 6t & y &= 4 - t \end{aligned}$$

Parametric equations for the line are $x = 8 + 6t$ and $y = 4 - t$.

Example 2 **Write an equation in slope-intercept form of the line whose parametric equations are $x = -3 + 4t$ and $y = 3 + 4t$.**

Solve each parametric equation for t.

$$\begin{aligned} x &= -3 + 4t & \qquad y &= 3 + 4t \\ x + 3 &= 4t & y - 3 &= 4t \\ \frac{x+3}{4} &= t & \frac{y-3}{4} &= t \end{aligned}$$

Use substitution to write an equation for the line without the variable t.

$$\begin{aligned} \frac{x+3}{4} &= \frac{y-3}{4} && \textit{Substitute.} \\ (x + 3)(4) &= 4(y - 3) && \textit{Cross multiply.} \\ 4x + 12 &= 4y - 12 && \textit{Simplify.} \\ y &= x + 6 && \textit{Solve for } y. \end{aligned}$$