4-6

Study Guide

Rational Equations and Partial Fractions

A **rational equation** consists of one or more rational expressions. One way to solve a rational equation is to multiply each side of the equation by the least common denominator (LCD). Any possible solution that results in a zero in the denominator must be excluded from your list of solutions. In order to find the LCD, it is sometimes necessary to factor the denominators. If a denominator can be factored, the expression can be rewritten as the sum of **partial fractions.**

Example 1 **Solve $\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$.**

$$6(x-2)\left[\frac{x+1}{3(x-2)}\right] = 6(x-2)\left(\frac{5x}{6} + \frac{1}{x-2}\right)$$ *Multiply each side by the LCD, 6(x − 2).*

$$2(x+1) = (x-2)(5x) + 6(1)$$

$$2x + 2 = 5x^2 - 10x + 6$$ *Simplify.*

$$5x^2 - 12x + 4 = 0$$ *Write in standard form.*

$$(5x-2)(x-2) = 0$$ *Factor.*

$$5x - 2 = 0 \qquad x - 2 = 0$$

$$x = \frac{2}{5} \qquad x = 2$$

Since x cannot equal 2 because a zero denominator results, the only solution is $\frac{2}{5}$.

Example 2 **Decompose $\frac{2x-1}{x^2+2x-3}$ into partial fractions.**

Factor the denominator and express the factored form as the sum of two fractions using A and B as numerators and the factors as denominators.

$$x^2 + 2x - 3 = (x-1)(x+3)$$

$$\frac{2x-1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$2x - 1 = A(x+3) + B(x-1)$$

Let $x = 1$.

$$2(1) - 1 = A(1+3)$$
$$1 = 4A$$
$$A = \frac{1}{4}$$

Let $x = -3$.

$$2(-3) - 1 = B(-3-1)$$
$$-7 = -4B$$
$$B = \frac{7}{4}$$

$$\frac{2x-1}{x^2+2x-3} = \frac{\frac{1}{4}}{x-1} + \frac{\frac{7}{4}}{x+3} \text{ or } \frac{1}{4(x-1)} + \frac{7}{4(x+3)}$$

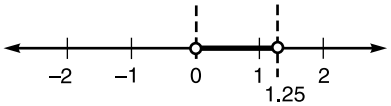
Example 3 **Solve $\frac{1}{2t} + \frac{3}{4t} > 1$.**

Rewrite the inequality as the related function $f(t) = \frac{1}{2t} + \frac{3}{4t} - 1$.

Find the zeros of this function.

$$4t\left(\frac{1}{2t}\right) + 4t\left(\frac{3}{4t}\right) - 4t(1) = 4t(0)$$

$$5 - 4t = 0$$

$$t = 1.25$$

The zero is 1.25. The excluded value is 0. On a number line, mark these values with vertical dashed lines. Testing each interval shows the solution set to be $0 < t < 1.25$.