5-7

Practice

The Ambiguous Case for the Law of Sines

Determine the number of possible solutions for each triangle.

1. $A = 42°, a = 22, b = 12$

2. $a = 15, b = 25, A = 85°$

3. $A = 58°, a = 4.5, b = 5$

4. $A = 110°, a = 4, c = 4$

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

5. $b = 50, a = 33, A = 132°$

6. $a = 125, A = 25°, b = 150$

7. $a = 32, c = 20, A = 112°$

8. $a = 12, b = 15, A = 55°$

9. $A = 42°, a = 22, b = 12$

10. $b = 15, c = 13, C = 50°$

11. ***Property Maintenance*** The McDougalls plan to fence a triangular parcel of their land. One side of the property is 75 feet in length. It forms a 38° angle with another side of the property, which has not yet been measured. The remaining side of the property is 95 feet in length. Approximate to the nearest tenth the length of fence needed to enclose this parcel of the McDougalls' lot.

NAME _____ DATE _____ PERIOD _____

5-7

Practice

The Ambiguous Case for the Law of Sines

Determine the number of possible solutions for each triangle.

1. $A = 42°$, $a = 22$, $b = 12$
1

2. $a = 15$, $b = 25$, $A = 85°$
0

3. $A = 58°$, $a = 4.5$, $b = 5$
2

4. $A = 110°$, $a = 4$, $c = 4$
0

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

5. $b = 50$, $a = 33$, $A = 132°$
none

6. $a = 125$, $A = 25°$, $b = 150$
$B = 30.5°$, $C = 124.5°$, $c = 243.7$;
$B = 149.5°$, $C = 5.5°$, $c = 28.3$

7. $a = 32$, $c = 20$, $A = 112°$
$B = 32.6°$, $C = 35.4°$, $b = 18.6$

8. $a = 12$, $b = 15$, $A = 55°$
none

9. $A = 42°$, $a = 22$, $b = 12$
$B = 21.4°$, $C = 116.6°$, $c = 29.4$

10. $b = 15$, $c = 13$, $C = 50°$
$A = 67.9°$, $B = 62.1°$, $a = 15.7$;
$A = 12.1°$, $B = 117.9°$, $a = 3.6$

11. ***Property Maintenance*** The McDougalls plan to fence a triangular parcel of their land. One side of the property is 75 feet in length. It forms a 38° angle with another side of the property, which has not yet been measured. The remaining side of the property is 95 feet in length. Approximate to the nearest tenth the length of fence needed to enclose this parcel of the McDougalls' lot.
about 312.1 ft

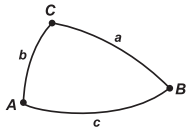
NAME _____ DATE _____ PERIOD _____

5-7

Enrichment

Spherical Triangles

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.



The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Example **Solve the spherical triangle given $a = 72°$, $b = 105°$, and $c = 61°$.**

Use the Law of Cosines.

$$\begin{aligned} 0.3090 &= (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A \\ \cos A &= 0.5143 \\ A &= 59° \end{aligned}$$

$$\begin{aligned} -0.2588 &= (0.3090)(0.4848) + (0.9511)(0.8746) \cos B \\ \cos B &= -0.4912 \\ B &= 119° \end{aligned}$$

$$\begin{aligned} 0.4848 &= (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C \\ \cos C &= 0.6148 \\ C &= 52° \end{aligned}$$

Check by using the Law of Sines.

$$\frac{\sin 72°}{\sin 59°} = \frac{\sin 105°}{\sin 119°} = \frac{\sin 61°}{\sin 52°} = 1.1$$

Solve each spherical triangle.

1. $a = 56°$, $b = 53°$, $c = 94°$
$A = 41°$, $B = 39°$, $C = 128°$

2. $a = 110°$, $b = 33°$, $c = 97°$
$A = 116°$, $B = 31°$, $C = 71°$

3. $a = 76°$, $b = 110°$, $C = 49°$
$A = 59°$, $B = 124°$, $c = 59°$

4. $b = 94°$, $c = 55°$, $A = 48°$
$a = 60°$, $B = 121°$, $C = 45°$